ESTIMATING THE VOLATALITY OF FOREIGN EXCHANGE RATE FOR BHUTAN- A GARCH APPLICATION

J.S. KHOKHAR*

This study investigates whether the foreign exchange rate of Bhutanese currency with US Dollar is volatile and predictable. The study considers 120 monthwise data on the exchange rate for the period 1997-2006, and finds the growth rate of the exchange rate to be a stationary series. The stationary series, which is given the functional form of an ARIMA(1,2) model, is volatile, and the volatility is represented by means of a GARCH(1,1) model. Moreover, the volatility is inversely connected with economic growth.

I- Introduction

Foreign trade is the backbone of the economic activities of Bhutan. Over the last decade, the ratio of exports to gross domestic product has varied from 0.21 to 0.32, and that of imports to GDP from 0.36 to 0.56. These relations imply that, in order to sustain a high rate of GDP, Bhutan should have a smooth functioning of its foreign trade. But, foreign trade generally depends on the exchange rate. If the exchange rate(XR) between Bhutan and rest of the world, defined as XR=Ngultrum (Bhutanese currency)/ one Dollar, fluctuates widely and erratically, Bhutan's export revenue and import bill will become uncertain, adversely affecting the monetary policy and economic growth of Bhutan. Over the last ten years, EX has shown a lot of variability. The co-efficient of variation of XR, which is (Standard deviation of XR/ mean of XR)x 100, has ranged between 0.52 and 3.73%. So, in this context, there are two relevant questions for investigation: Is XR volatile?, and, if it is, can the volatility of XR be estimated or predicted?

To find empirical answers to those two questions in the framework of a GARCH (generalised autoregressive conditional heteroscedasticity) model [pp. 118-127, Enders(2004)] is the main purpose of this article.

For the purpose, the rest of the article is organised into five sections: II- Background, III- Data, IV- GARCH Methodology, V- Results, and VI- Conclusion.

* Reader in Economics, Shri Ram College of Commerce, University of Delhi, Delhi-110007.

II- Background

This section is made up with two parts: Bhutan and its economy, and a brief note about exchange rate volatility.

2.1- Bhutan and its Economy

Bhutan is a small kingdom situated in eastern Himalayas, with its latitude varying from 26.7^0 North to 28.4^0 North, and longitude from 88.7^0 East to 92.2^0 East. The Kingdom covers an area of 46,620 sq.kms. At the maximum, the area has its north-south distance of 170 kms and east-west distance of 300 kms. The Kingdom, which is landlocked between China and India, is bordered by the Tibetan region of China and the Indian states of Sikkim, West Bengal, Assam, and Arunchal Pradesh.

Bhutan is an extremely mountainous country with its altitude ranging from over 7,000 metres down to less than 100 metres. As a result, the country has several steep mountains, deep gorges, and fast flowing rivers. Most of these rivers run from north to south, cutting several thousand metres deep and V-shaped valleys through the mountains in such a manner that hardly any flat ground is left on the valley floor at all, and providing Bhutan with a potential of 30,000 MW of hydro electricity.

The economy of Bhutan has two main features. One, the economic activity of Bhutan is predominantly agrarian, with crop production and livestock rearing accounting for the bulk of the total output. The agricultural sector provides the main livelihood for 90% of the population, and accounts for 36% of GDP. Much of the agricultural activity is subsistence-based and takes place outside the exchange sector. This way, many goods and services in the rural areas are not monetised. The other feature is that the economy is closely aligned with India through a strong trade. The direction of trade reveals that Bhutan sells 87.6% of its exports to India, and buys 75.1% of its imports from India. These trade links result from the free trade arrangements under which there are no restrictions on trade and payments between the two countries.

In connection with trade, it shall be mentioned that the trade balance of Bhutan with India and all other countries taken together has been in deficit since 1997, the first year of our study period; but the balance of payments of Bhutan has always been in surplus since 1997. The capital inflows into Bhutan on account of foreign tourists, direct foreign investment, foreign grants, etc. have been of such magnitudes that outweighed all the payment obligations of Bhutan to other countries.

2.2- A brief Note about Exchange Rate Volatality

Perhaps, the most comprehensive and detailed survey of the various statistical and econometric models used to measure and forecast the volatility of an economic variable is available in the study by Poon and Granger(2003).

The study concentrates on two questions: Is volatility forecastable? If it is, which method will provide the best forecasts? To consider these questions, the study divides all the various models that make volatility forecasts into the following four categories:

1. HISVOL, historical volatility models, includes random walk, historical averages of squared returns or absolute returns. Belonging to this category are also time series models based on historical volatality using moving averages, exponential weights, autoregressive models, or even fractionally integrated autoregressive absolute returns, for example.

2. GARCH covers any member of the ARCH, GARCH, EGARCH, and so forth families (Chapter 3, Enders, op.cit.).

3. ISD, option standard deviation, is based on the Black Scholes model and various generalizations; and

4. SV consists of stochastic volatility models for making forecasts.

The study includes 93 paper contributions. Their overall ranking suggests that ISD provides the best forecasting with HISVOL and GARCH roughly equal, although possibly HISVOL does somewhat better in the comparisons. The success of the implied volatility should not be surprising as these forecasts use a larger and more relevant information set than the alternative methods as they use option prices. They are also less practical, not being available for all assets.

Among the 93 papers, seventeen studies compared alternative versions of GARCH. It is clear that GARCH dominates ARCH. In general, models that incorporate volatility asymmetry such as EGARCH and GJR-GARCH perform better than GARCH. But, certain specialised specifications such as fractionally integrated GARCH (FIGARCH) and regime switching GARCH(RSGARCH) do better in some studies. However, it seems clear that one form of study that is included is conducted just to support a viewpoint that a particular method is useful.

In the context of our approach towards forecasting the volatility of the foreign exchange rate between Bhutan and United States, we shall mention two more studies: Caporale and Mckiernan(1996) and Singh(2002)

Caporale and Mckiernan(1996) investigate the relationship between output variability and economic growth by using a GARCH-M model with the data on industrial production in post-war Great Britain. The model consistently parameterises the conditional variance of output growth. The results reveal a positive relationship between variability and growth rate of output.

Singh(2002) estimates the generalised autoregressive conditional heteroscedasticity(GARCH) model for a comprehensive set of both weighted(export and trade) as well as un-weighted(official and black market) real exchange rate series in India. The study finds the evidence of dimensionally weak and statistically insignificant autoregressive conditional heteroscedasticity(ARCH) effects as compared to GARCH effects in almost all the exchange rate series. The estimates of the GARCH model are sensitive to the measure of exchange rate used. Besides, the GARCH effects remain invariant to the choice of sample period, and the evidence points towards the regime neutrality of exchange rate volatility in India.

III- Data

We have considered 120 monthwise observations on the exchange rate between Bhutan and United States, XR=Ngultrum/US Dollar, for ten years from 1997 to 2006. These observations were collected from two issues(December- 1994 and December-2006) of the Selected Economic Indicators, and the April 2007 issue of the Monthly Statistical Bulletins. Both the documents are prepared and published by the Royal Monetary Authority of Bhutan, Royal Government of Bhutan, Thimphu. All the 120 observations are mentioned in Table-1, and, in an attempt to know the pattern of the exchange rate over the period of our study, we have plotted these observations in Figure-1, where t=1 for the first month of 1997, t=2 for the second month of 1997, and so on till t=120 for the last month of 2006.

Table-1
Monthly Data on Exchange Rate, XR, Ngultrum/US Dollar

Year - Month	XR	Year - Month	XR	Year- - Month	XR	Year/ - Month	XR	Year/ - Month	XR
1997		1999-		2001-		2003-		2005-	
-Jan.	35.87	Jan	42.51	Jan	46.54	Jan	47.93	Jan	43.76
Feb.	35.89		42.46		46.52		47.74		43.68
March	35.87		42.44		46.62		47.65		43.69
April	35.82		42.73		46.79		47.38		43.74
May	35.81		42.77		46.92		47.08		43.49
June	35.81		43.14		47.01		46.72		43.58
July	35.74		43.29		47.14		46.23		43.54
August	35.93		43.46		47.13		45.93		43.63
Sept.	36.42		43.54		47.65		45.85		43.92
Oct.	36.23		43.45		48.02		45.39		44.82
Nov.	37.15		43.40		48.00		45.52		45.73
Dec.	39.22		43.49		47.92		45.59		45.64
1998		2000		2002		2004		2006	
-Jan	39.36	-Jan	43.55	-Jan	48.34	-Jan	45.46	-Jan	44.40
	38.91		43.61		48.69		45.27		44.33
	39.50		43.59		48.74		45.02		44.48
	39.65		43.64		48.92		43.93		44.95
	40.37		43.97		49.00		45.25		45.41
	42.25		44.69		48.96		45.51		46.06
	42.51		44.78		48.76		46.04		46.46
	42.76		45.69		48.59		46.34		46.54
	42.52		45.89		48.44		46.10		46.12
	42.35		46.35		48.37		45.78		45.47
	42.38		46.78		48.38		45.13		44.85
	42.55		46.75		48.14		43.98		44.64

Figure-1: Pattern of exchange rate

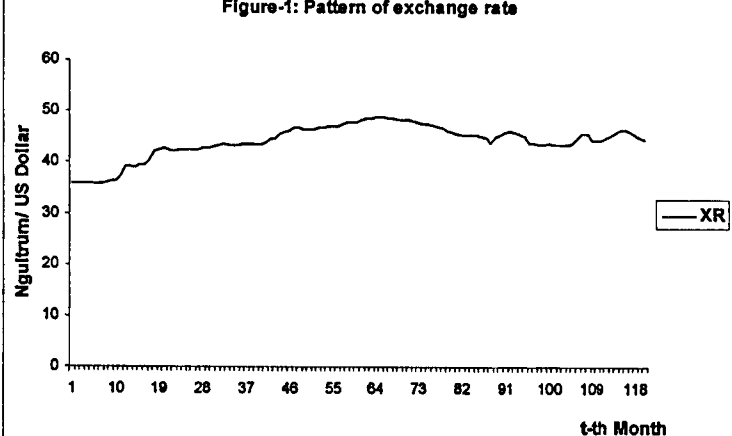

Figure-1 indicates that the exchange rate has not been changing smoothly since the first month of 1997. The exchange rate has registered several upward and downward swings. But, in order to avoid a spurious measure of the volatility of exchange rate, we need to work with a stationary variable which could be XR variable itself or some other form of XR variable.

A variable is stationary if its series of data does not have a unit root. In order to know whether XR variable or some other form of the variable has a unit root, we apply Dickey-Fuller test to the series of XR variable; first-differenced values of XR variable, (ΔXR); natural log of XR variable, (LXR); and first-differenced values of the natural log of XR variable, (ΔLXR). The test has been applied to each series under the null hypothesis that the series has a unit root or is non-stationary. The results from the test are reported in the following table:

Table-2
Unit Root Results

Variable	Dickey-Fuller test
XR	(1.529)
ΔXR	(-7.306)*
LXR	(1.794)

Note: (a) The figure in a bracket stands for the t-value of the regression coefficient which is supposed to be zero under the null hypothesis,

(b) * means the t-value is significant at 1% level, and

(c) EViews- 3.1, an econometric software, was used.

Table-2 reveals that the null hypothesis of unit root is rejected in the cases of ΔXR and ΔLXR variables, implying that these variables have stationary series of data. We prefer ΔLXR to ΔXR variable for the simple reason that multiplying ΔLXR by 100 we get the growth rate of XR in percentage, GXR. Moreover, for the purpose of analysing the volatility of GXR, we have plotted all the 119 observations on GXR in following figure:

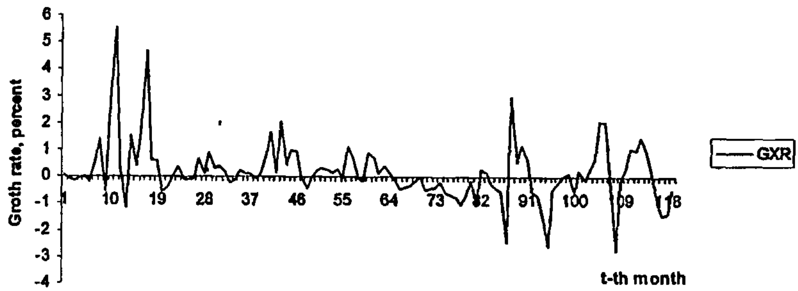

Figure-2: Volatility of GXR

Figure-2 shows that GXR has not registered a uniform rise or fall over the last decade. It has had wide variations, and it is this volatility of GXR that we are going to capture in an algebraic model in the next section.

IV- GARCH Methodology

We apply GARCH, generalised autoregrssive conditional hetroscedasticity, developed by Bollerslev(1986) as an extention to Engle's(1982) ARCH, autoregressive conditional heteroscedaticity, to the volatality of the growth rate of exchange rate, GXR, which has already been proved to be a stationary series, in three stages.

At the first stage, we try to represent the graph of GXR in terms of an algebraic equation. For this purpose, we employ Box-Jankins(1976) strategy by incorporating various combinations of autoregressive(AR) and moving average(MA) schemes up to the second order. That way, we work out eight equations: AR(1), AR(1)-AR(2), MA(1), MA(1)-MA(2), AR(1)-MA(1), AR(1)-MA(1)-MA(2), AR(1)- AR(2)- MA(1), and AR(1)-AR(2)-MA(1)-MA(2). Out of these equations, we pick up the one which has the least value of Akaike Information Criterion(AIC) or Schwarz Criterion(SC) and has no serial correlation. To check for the absence of serial correlation, we are using the Ljung-Box Q-statistic[pp. 812-13, Gujarati(2003)].

At the second stage, we check whether the residuals of the chosen equation show ARCH or GARCH effects. This will be done by means of F and χ^2 tests(pp. 118-20, Enders, op. cit.). If the tests do not indicate such effects, we stop here and retain the chosen equation for making forecasts about GXR. On the other hand, if the test reveals such effects, we move on to the next stage.

At this stage, we consider the residuals which show ARCH or GARCH effects. The series of these residuals($e_1, e_2, \ldots, e_T$) is expected to behave as follows:

$$e_t = v_t \sqrt{h_t} \quad \ldots \qquad (1)$$

$$t = 1, 2, \ldots, T$$

and

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \quad \ldots \qquad (2)$$

where v_t is a white noise with $E(v_t)=0$, and $Var(v_t) = E(v_t^2) = 1$. Moreover, $E(e_t)=0$, and $Var(e_t)=E(e_t^2)= h_t$.

By assigning various lag lengths to p and q, we get several estimations of equation (2), and will choose that computed equation which does not have ARCH or GARCH effects. But, if there is more than one such an equation, then we will pick up that equation which has the least value of the following expression:

$$L = \sum_{t=1}^{T} [e_t^2 - h_t]^2 \quad \ldots \qquad (3)$$

where L= loss function, which is the sum of the differences between actual residuals, e_t^2, and the fitted value of the conditional variance, h_t.

Equation(3) or loss fuction as a selection criterion is suggested by Bollerslev, Engle, and Nelson(1994), who say that the statistical properties of AIC or SC as a conventional criterion of selection are unknown in the context of ARCH models.

V- Results

For the first stage of our methodology, we have estimated all the eight ARIMA equations, which are presented in the following table:

Table- 3
Results from ARIMA Model for GXR Variable

Model	Constant	AR(1)	AR(2)	MA(1)	MA(2)	R^2	AIC	SC	Ljung-Box Q-statistic
ARIMA(1, 0)	0.182	0.349	-	-	-	0.121	2.892	2.939	18.685
		(1.267)	(4.001)*						
	0.929@								
ARIMA(2, 0)	0.186	0.365	-0.047	-	-	0.123	2.916	2.986	19.067
		(1.339)	(3.899)*		(-0.502)				
	0.896@								
ARIMA(0, 1)	0.183	-	-	0.372	-	0.125	2.879	2.926	18.472
		(1.439)			(4.331)*				0.934@
ARIMA(0, 2)	0.183	-	-	0.375	0.014	0.125	2.896	2.966	18.531
		(1.414)			(4.040)*	(0.150)			0.912@
ARIMA(1, 1)	0.184	0.078	-	0.299	-	0.125	2.905	2.975	18.512
		(1.388)	(0.309)		(1.238)				0.913@
ARIMA(1, 2)	-0.141	0.970	-	-0.641	-0.339	0.168	2.871	2.965	17.807
		(-0.557)	(33.753)*			(-6.507)*		(-4.005)*	0.909@
ARIMA(2, 1)	0.185	-0.16	0.108	0.540	-	0.130	2.926	3.021	17.987
		(1.331)	(-0.218)	(0.354)	(0.750)				0.904@
ARIMA(2, 2)	0.185	0.144	0.145	0.234	-0.145	0.130	2.942	3.06	17.810
		(1.265)	(0.087)	(0.441)	(0.142)	(-0.296)			0.883@

Note: Brackets contain t-values,
* means significant at 1% level,
Q- statistic is calculated at the 40th lag, number of observations divided by 4, and
@ means probability.

From the eight equations of the table, we have to pick up one equation which is free from serial correlation but has the least magnitude of AIC or SC variable.

Table-3 indicates that all the equations bear no serial correlation. The value of Q-statistic is insignificant with a very high probability, implying that the null hypothesis of no serial correlation is not rejected. The table also shows that AIC takes its smallest value of 2.871 for ARIMA(1,2) equation, and SC 2.926 for ARIMA(0,1) equation. So, we can choose either equation, but we prefer ARIMA(1,2) equation because it carries a higher value of R^2, co-efficient of determination, and all its three explanatory variables are statistically significant at 1% level , too.

Now, our next step is to test whether ARIMA(1,2) equation contains any ARCH or GARCH effects. For the purpose, we calculate F and Chi-square tests under the null hypothesis that the residuals of our equation do not show any ARCH or GARCH effects. The results from these two tests are presented as follows:

Table- 4
ARCH Test

Test	Value	Probability
1. One-year lag F-statistic	8.560	0.004
X^2-statistic (Obs*R-squared)	0.106	0.004
2: two-year lag F-statistic	4.196	0.017
X^2-statistic (Obs*R-squared)	8.019	0.018

Table-4 indicates that both the test-statistics are significant at 1% for a lag-period of one year, implying that ARCH or GARCH effects are present up to one-year lag period. As a result, we shall work out two GARCH models: GARCH(1,0) and GARCH(1,1).

The computed forms of both the GARCH models are mentioned as follows:

1. GARCH(1,0)- Model

$$GXR_t = \underset{(1.068)}{0.089} + \underset{(3.768)^*}{0.462}\,AR(1) - \underset{(-0.845)}{0.097}\,MA(1) +$$

$$\underset{(5.222)^*}{0.211}\,MA(2) \quad \ldots \qquad (4.1)$$

$$h_t = \underset{(3.928)^*}{9.154} + \underset{(4.018)^*}{1.368}\,e^2_{t-1} \quad \ldots \qquad (4.2)$$

Loss function(L) = 2160.84

Note: * means significant at 1% level.

2. GARCH(1,1)-Model

$$GXR_t = 0.120 + 0.732\,AR(1) - 0.186\,MA(1) -$$

(1.002) (5.721)* (-1.049)

0.085 MA(2) . . . (5.1)
(-0.877)

$$h_t = 0.119 + 1.211\, e^2_{t-1} + 0.088\, h_{t-1} \quad \ldots \quad (5.2)$$
(2.517)* (3.878)* (0.945)

LOSS FUNCTION(L) = 1384.62

Note: * means significant at 1 % level.

As the loss function is smaller for GARCH(1,1) model, we take up this model for further analysis, and describe its equations (5.1) and (5.2).

Equation (5.1) has only one significant explanatory variable. It is AR(1)= GXR_{t-1}, the previous growth rate of exchange rate, which has a positive coefficient. That is to say, a rise(fall) in the previous growth rate of exchange rate will raise or reduce the current growth rate of exchange rate. But, neither MA(1)= e_{t-1}, one- year past residual; nor MA(2)= e_{t-2}, two-year past residual, affects the current rate of exchange rate significantly.

Equation (5.2) reveals that the conditional variance, h_t, is only affected by the square of the previous residual. The coefficient of the square of the previous residual is positive, implying that a fall(rise) in the squared value of the past residual will reduce or increase the current value of the conditional variance, But, the past value of the conditional variance has no significant impact on the current value of the conditional variance.

There are still two more features of equation (5.2) to be mentioned. One, the coefficient of h_{t-1} is strictly between 0 and 1, meaning whatever the little impact h_{t-1} has on h_t will vanish in the long run. Another, the coefficient of e^2_{t-1} is greater than 1, indicating that the squared value of the past residual will influence that current value of the conditional variance in an explosive manner.

However, equation (5.2) can be used in its entire form for the purpose of forecasting the volatility or conditional variance of the exchange rate. The graph of the conditional variance is presented in the following figure.

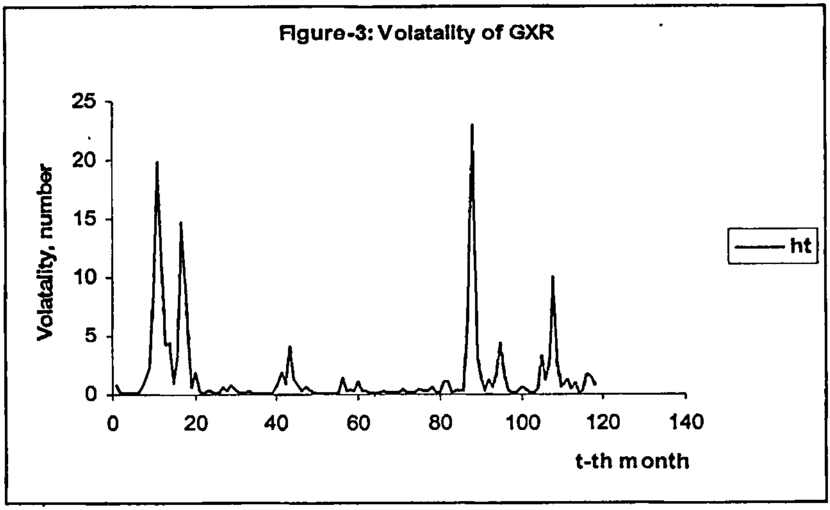

Figure-3 forecasts that the volatility of GXR will behave in the form of a bathtub, remaining at a very high level at the both ends of a period of ten years, and changing slightly at a very level over the intervening months.

VI- Conclusion

Our study brings out three points for conclusion. One, the growth rate of the foreign exchange between Bhutanese currency and US Dollar forms a stationary series. It does not have a unit root. Another, the growth rate of the foreign exchange is volatile, and the volatility can be estimated in the framework of a GARCH(1,1) model. Finally, the volatility, which behaves in the shape of a bathtub, is negatively connected with economic growth. At a higher growth rate of GDP, the variations in the growth rate of foreign exchange are smaller. However, the validity of our conclusion can not be ascertained unless a few more similar studies are conducted.

REFERENCES

Bollerslev, T., 1986. Generalized Autoregressive Conditional Hetroscedasticity, *Journal of Econometrics*, 31, pp. 307-27.

Bollerslev, T., Engle, R.F., and Nelson, D.B., 1994. ARCH Models, in *Handbook of Econometrics*, Vol. IV, edited by R.F. Engle and D.L. McFadden, North-Holland, pp. 2959-3038.

Box, G., and Jenkins, G.,1976. *Time Series Analysis, Forecasting, and Control*, San Francisco, Holden Day.

Bredin, D., and Fountas, S., 2005. Macroeconomic Uncertainty and Macroeconomic Performance: Are they related?, Manchester School, University of Manchester, Vol. 73(s1), pp. 58-76.

Caporale, T., McKiernan, B., 1996. The Relationship between Output Variability and Growth: evidence from post war UK data, *Scottish Journal of Political Economy*, 43, pp. 229-36.

Comin, D., and Mulani, S., 2003. *Diverging Trends in Macro and Micro Volatility: facts., an Economic Research Report*, C.V. Starr Centre for Applied Economics, Department of Economics, Faculty of Arts and Science, New York University. New York.

Enders, W.,2004. *Applied Econometric Time Series*, John Wiley & Sons, Inc., Singapore.

Engle, R.F., 1982. Autoregressive Conditional Hetroscedasticity with Estimates of the Variance of the United Kingdom Inflation, *Econometrica*, 50, pp. 987- 1007.

EViews-3.1., 1998. *Quantitative Micro Software*, USA.

Gujarati, D.N., 2003. *Basic Econometrics*, Tata McGraw-Hill Publishing Company Limited, Fourth Edition, New Delhi.

Henry, O.T., and Olekalns, N., 2002. The Effect of Recession on the Relationship between Output Variability and Growth, *Southern Economic Journal*, 68, pp. 683- 92.

Lastrapes, W.D., 1989. Exchange Rate Volatality and U.S Monetary Policy: An ARCH application, *Journal of Money, Credit, and Banking*, 21, pp. 66-77.

Monthly Statistical Bulletin, 2007. *Royal Monetary Authority of Bhutan*, Royal Government of Bhutan, Thimphu, April.

Pindyck, R.S., and Rubinfeld, D.L., 1998. *Econometric Models and Forecasts*, Irwin McGraw-Hill, New York.

Poon, S.H., and Granger, C.W.J., 2003. Forecasting Volatality in Financial Markets: A Review., *Journal of Economic Literature*, Vol XLI, pp. 478-539.

Pozo, S., 1992. *Conditional Exchange-Rate Volatality and Volume of International Trade: Evidence from the early 1900s*, LXXIV, pp. 325-29.

Selected Economic Indicators., 2001 & 2006. *Royal Monetary Authority of Bhutan*, Royal Government of Bhutan, Thimphu, December.

Sengupta, A., and Sfeir, R., 1996. *Modelling Exchange Rate Volatality*, Series 11-96, University of California, Department of Economics, Santa Barbara, CA, USA.

Singh, T., 2002. On the GARCH Estimates of Exchange Rate Volatility in India, *Applied Economics Letters*, 9, pp. 391-95.