STUDY OF COLLECTIVE BEHAVIOUR IN STOCK MARKET AS SOCIAL PHASE TRANSITION

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In this paper we have investigated a non-linear model for stochastic dynamics of stock market as social phase transition. Two-body interactions arising through imitative behavior of "investors", those who parts away with the surplus cash in hand for anticipated higher return in future and "noise trader", those who act on little information, even if they think they "know", render the model non-linear. Two competing sub-processes: global cooperation among noise traders causing collective behavior due to various economic and psychological reasons and investors' heterogeneity in framing of information, cognitive errors, self control and lack thereof, regret in financial decision making, leads to the manifestation of threshold effect and the onset of market crash. We witness a second order phase transition when a certain parameter β approaches a critical value β_c. As changes from $\beta < \beta_c$ to, $\beta > \beta_c$ system exhibits a cooperative behavior and transits from a state, possessing geometric distribution to a state with quasi-Poissonian distribution, indicating that the system is going to acquire a new structure replacing the old one. The situation is quite analogous to that happening in laser physics. Market crashes are unavoidable part of our world that affects the lives of millions of people around the globe. In this analysis we have tried to develop early warning system that lead to effective crisis management and damage control.

Keywords: Two Body Interaction, Cooperative Behavior, Threshold Effect, Second Order Phase Transition.

INTRODUCTION

These days there is large supply of literature regarding "psychology of investor" thus it is evident that the search continues to find the proper balance of traditional finance, behavioral finance, behavioral economics, psychology, and sociology. This area of research is an interdisciplinary approach including scholars from the social sciences and

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business schools. In this paper we have investigated an interactions arising through interpersonal contacts among investors due to which system exhibits a cooperative behavior and acquire a new structure replacing the old one. Analysis may provide with early warning to help in crisis management. A market crash occurring simultaneously on most of the stock markets of the world as witnessed in October 1987 would amount to the quasi-instantaneous evaporation of trillions of dollars. In values of January 2001, a stock market crash of 30% indeed would correspond to an absolute loss of about 13 trillion dollars. From the opening on October 14, 1987 through the market close on October 19, major indexes of market valuation in the United States declined by 30 percent or more. Furthermore, all major world markets declined substantially in the month, which is itself an exceptional fact that contrasts with the usual modest correlations of returns across countries and the fact that stock markets around the world are amazingly diverse in their organization (Barro et al., 1989). There are growing empirical evidences of the existence of herd or "crowd" behavior in speculative markets (Shiller, 2000). Herd behavior is often said to occur when many people take the same action, because some mimic the actions of others. Keynes (1936) argued that stock prices are not only determined by the firm's fundamental value, but, in addition, mass psychology and investors' expectations influence financial markets significantly. The imitative behavior belongs to a very general class of stochastic dynamical models developed to describe interacting elements, particles,

agents in a large variety of contexts, in particular in physics and biology (Liggett, 1997). Same characteristics, in particular apparent coordinate buying and selling periods, leading eventually to several financial crashes. These features are: a system of traders who are influenced by their "neighbors", local imitation propagating spontaneously into global cooperation, prices related to the properties of this system etc. As it evident through different studies that crash is most likely when the locally imitative system goes through a critical point. In Physics, critical points are widely considered to be one of the most interesting properties of complex systems. A system goes critical when local influences propagate over long distances and the average state of the system becomes exquisitely sensitive to a small perturbation, i.e. different parts of the system become highly correlated. Another characteristic is that critical systems are self-similar across scales, at the critical point, an ocean of traders who are mostly bearish may have within it several continents of traders who are mostly bullish, each of which in turns surrounds seas of bearish traders with islands of bullish traders; the progression continues all the way down to the smallest possible scale: a single trader (Wilson, 1979). Intuitively speaking, critical self-similarity is why local imitation cascades through the scales into global coordination. Critical points are described in mathematical parlance as singularities associated with bifurcation and catastrophe theory.

Though the idea of evolution was first introduced in physics in the 19th century through the so-called second law of thermodynamics by Kelvin and Clausius independently which states that "the entropy of the universe (a measure of disorder) is increasing" (Nicols and I. Prigogine, 1977). This idea of evolution was propounded almost simultaneously in the 19th century in biology (Darwin, 1859) and enunciated with different interpretation in sociology (Spencer, H. 1904). The quintessence of the idea of evolution is that no system whether physical, biological or social is structurally stable. New orders emerge through fluctuations.

THE MODEL

Since in the real world problems micro-scopic activities result into macro-scopic manifestation, we shall formulate and quantify the overall dynamic behaviour of the process as phenomenological, super-imposed with statistical fluctuations. In statistical mechanics, phase transitions where a very small parameter change leads to a chaotic change in the system's macroscopic properties have been extensively studied (Stanley, 1987; Anderson, 1997). Thus, in searching for an explanation for market crashes, it is natural to look for analogies between stock markets and statistical mechanics systems. Indeed, stock market systems have some fundamental features in common with statistical mechanics systems, such as a system of spins in a magnet. Both systems are composed of many interacting elements. (Investors or spins) that have an inclination to conform with one another and is driven by the magnetic force. In case of stock market investors also have various economic and psychological reasons that lead to collective behavior in stock market as social phase transition (Levy, M., 2005). The latest trends show that the market crash evolves through a combination of following sub-processes:

(i) awareness of investor through an external and/or internal source without getting influenced by noise traders.

(ii) global cooperation among noise traders or imitative behavior of investors causing collective behavior due to various economic and psychological reasons.

(iii) degree of investors' heterogeneity in framing of information, cognitive errors, self control and lack thereof, regret in financial decision making.

Let n(t) is the value of index at any time t and $\mathbf{p_n(t)}$ be the probability of index value at time t in the system of overall capacity N (maximum finite level of index). The evolution of the process may be looked upon as a birth-and-death process, governed by the transition probabilities in the time interval (t, t+t):

$$\mathbf{Pr\ [n \rightarrow n + 1] = \lambda_n \delta t + o(\delta t), \quad n \geq 0,} \tag{1.1}$$

$$\mathbf{Pr\ [n \rightarrow n - 1] = \mu_n \delta t + o(\delta t), \quad n \geq 1,} \tag{1.2}$$

where,

$$\lambda_n = \mathbf{a(N\text{-}n) + bn\,(N\text{-}n), \qquad \mu_n = cn} \tag{1.3}$$

are the birth and death rate constants. Here 'a' is the measure of intensity investor awareness through external and/or internal source, 'b' a measure of intensity of inter-personal contacts, and 'c' a measure of the heterogeneity of investors. As the initial condition, we suppose the presence of initial value of index n_0 the process. The conservation of probability yields the so-called master equation [ME] of the process (Parzen, 1962, van Kampan, 1981, Cox and Miller,1965):

$$\frac{d(p_n(t))}{dt} = \lambda_{n-1}\, \mathbf{p_{n-1}}\,(\mathbf{t_1}) + \mu_{n+1}\, \mathbf{p_{n+1}(t)} - (\mu_n + \lambda_n)\, \mathbf{p_n(t)}, \tag{2.1}$$

satisfying the initial condition

$$pn(0) = \delta n, n_{0,} \tag{2.2}$$

where δij is the Kronecker delta function defined by

$$\delta ij \begin{cases} 1, & \text{if} \quad i \neq j \\ 0, & \text{if} \quad i \neq j \end{cases} \tag{2.3}$$

in terms of the translation or shift operator E, defined by

$$E^{\pm 1} f(n) = f\,(n \pm 1)$$

The ME can be expressed in the form

$$\frac{d(p_n(t))}{dt} = \mathbf{(E\text{-}1)[\,\mu_n p_n(t) - E^{-1}(\lambda_n p_n(t))]} \tag{3}$$

Solution of the Model

An exact solution of equation (3), valid for all times, is mathematically intractable, however, an asymptotic distribution, as t, can be easily obtained. Thus in the steady state of the system, the master equation (3) reduced to:

$$(E-1)[\ \mu_n\, p_n(t)\ - E^{-1}(\lambda_n\, p_n(t))] = 0 \tag{4.1}$$

where,

$$p_n = p_n(\infty) \tag{4.2}$$

Since (E-1) is not the null operator, equation (4) leads to:

$$\mu_n\, p_n\ -\ \lambda_{n-1}\, p_{n-1}\ =\ C, \text{ a constant} \tag{5}$$

The constant C signifies the net probability flow from state n to state n-1. Since n= 0, is a natural boundary, we note that C vanishes, and equation (5) leads to the identity for one-step process:

$$\mu_n\, p_n = \lambda_{n-1}\, p_{n-1}, \tag{6}$$

in conformity with the 'principle of detailed balancing' in physical science (Medhi, 1982, Kannan 1983). Using the normalization condition, we get

$$p_n = \prod_{i=0}^{n-1} (\lambda_i / \mu_{i+1}) / \left[\sum_{n=0}^{N} \prod_{i=0}^{n-1} (\lambda_i / \mu_{i+1}) \right] \tag{7}$$

For the sake of clarity and brevity, we set

$$a/b = \alpha,\ \ Nb/c = \beta,\ \ \text{and } c/b = \gamma, \tag{8}$$

A close examination of (8) shows that α is an index of relative strength of the awareness as compared to that of personal contacts, β is the number of interpersonal contacts that an activist makes during his active period. Since b^{-1} is the mean duration of inter-personal contacts and γ is a measure relative strength of the degree of hetrogeneity as compared to that of personal contacts. Before carrying out stochastic analysis of the model, we shall examine the role of non-linearity which is responsible for bringing out phase transition (Pathria, 1972).

The Role of Non-linearity

The non-linearity of the model arises from the interactive process operating in the system. We begin with deterministic analysis. The dynamics of the model is governed by the deterministic equations:

$$dn(t)/dt = a(N-n) + bn(N-n) - cn, \tag{9}$$

can be written as

$$\frac{d(n/N)}{d(bNt)} = \frac{\alpha}{N} + \left(1 - \frac{1}{\beta} - \frac{\alpha}{N}\right)\frac{n}{N} - \left(\frac{n}{N}\right)^2 \tag{10}$$

As $\alpha = O(1)$, (10) can be approximated to

$$\frac{d(n/N)}{d(bNt)} \cong \left(1 - \frac{1}{\beta}\right)\frac{n}{N} - \left(\frac{n}{N}\right)^2 \tag{11}$$

If $\beta<1$, and the non-linear term $- bn^2$ is absent in Eqn. (10), the only equilibrium solution would be)

$$(n/N)_\infty = 0 \qquad (\text{for } \beta \le 1) \tag{12}$$

However, in the presence of the non-linear term, we have another branch of the solution:

$$(n/N)_\infty = \left(1 - \frac{1}{\beta}\right) \qquad (\text{for } \beta > 1) \tag{13}$$

Thus it turns out that as β increases gradually from the value less than 1 to a value greater than 1, the cooperative effects arising from the interpersonal contacts allow the system to transit from solution (12) to solution (13), (Sharma, Pathria, Karmeshu 1982, Karmeshu, Pathria, 1979), and obviously at β=1, the two solutions coalesce. Thus $\beta =_c = 1$, is the "critical" or the so-called "threshold" value of the parameter at which bifurcation of the solution takes place (Sattinger, 1973), (see Fig. 1). This transition is similar to the "lasing transition" (Thyagarajan, Ghatak, 1981, Tarasov, 1983).

The order parameter $(n/N)_\infty$ is a continuous function of the control parameter β, for $\beta<1$ and $\beta>1$, and has a discontinuity of second kind at $\beta=1$. Thus bifurcation of the solution takes place at this point. The dotted curve represents smooth transition from one branch to the other branch through fluctuations, high-lighting the role of stochasticity in the interval $(1-\varepsilon, 1+\varepsilon)$, $\varepsilon = O(N^{-1/2})$.

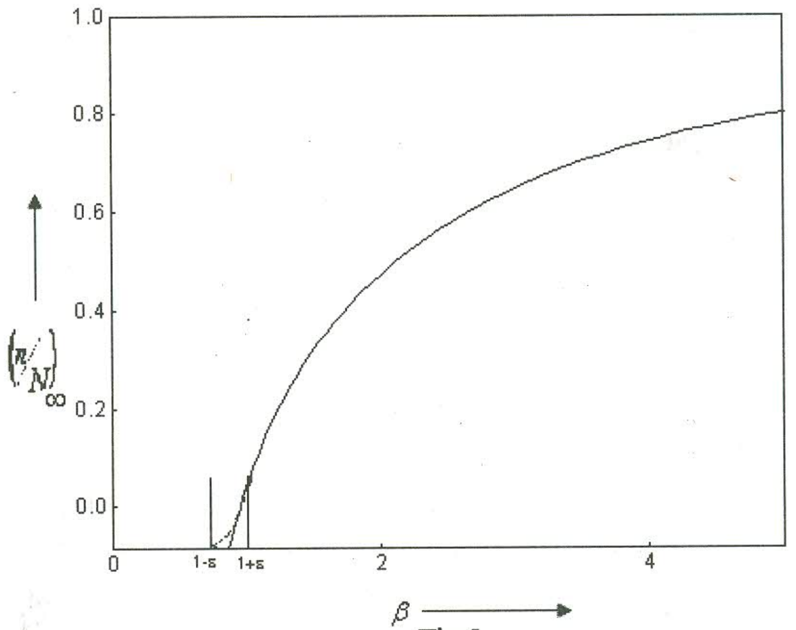

Fig-1

Stochastic Analysis of the Model with $\alpha = 1$

The equilibrium probability distribution given by relation (8) reduces to

$$p_n = \binom{N}{n} \gamma^{N-n} (n!) \Big/ \sum_{n=0}^{N} \binom{N}{n} \gamma^{N-n} (n!) \tag{14}$$

Since,

$$n! = \Gamma(n+1) = \int_0^\infty x^n e^{-x} dx,$$

The denominator in (14) can be expressed as

$$\begin{aligned} \sum_{n=0}^{N} \binom{N}{n} \gamma^{N-n} \int_0^\infty x^n e^{-x} dx &= \int_0^\infty e^{-x} (\gamma + x)^N dx \\ &= \gamma^{N+1} \int_0^\infty e^{-\gamma t} (1+t)^N dt \\ &= \gamma^{N+1} K_N(\gamma), \end{aligned} \tag{15}$$

where,

$$K_N(\gamma) = \int_0^\infty e^{-\gamma t} (1+t)^N dt. \tag{16}$$

Integration by parts leads to

$$K_N(\gamma) = \gamma^{-1}\left[1 + NK_{N-1}(\gamma)\right] = \gamma^{-2}\left[\gamma + N + N(N-1)K_{N-2}(\gamma)\right] \quad (17)$$

This recursive relation for $K_N(\gamma)$, readily yields

$$E(n) \equiv N - \gamma(1-p_0) \quad (18.a)$$

$$Var(n) \equiv \sigma_n^2 = \gamma^2 p_0(1-p_0) + \gamma(1-p_0) - N\gamma\, p_0, \quad (18.b)$$

where,

$$p_0 = \left[\gamma K_N(\gamma)\right]^{-1}. \quad (18.c)$$

Obviously, the stochastic behavior of the given system is primarily governed by the manner in which the function $K_n(\gamma) = K_N(N/\beta)$ varies with β. We discuss the situation when β is significantly above 1 and significantly below 1. The close neighborhood of $\beta = 1$ of size $O(N^{-1/2})$ is the critical region. Here we shall deal with two cases:

Case (i) significantly below 1:

It can be easily seen that for $\beta << 1$, the probability distribution given by (14) reduces to a geometric distribution:

$$p_n = (1-\beta)\beta^n, n = 0,1,2,..... \quad (19)$$

with,

$$\bar{n} = \beta/(1-\beta) \text{ and } \sigma^2{}_n = \beta/(1-\beta)^2 \quad (20)$$

we note here that and $\bar{n}$ and σ^2_n both are of O(1), and satisfy the relation

$$\sigma^2/\bar{n}^2 = 1 + \frac{1}{\bar{n}} = \frac{1}{\beta} \quad (21)$$

Case (ii) significantly above 1:

Replacing the integrand in (16), by its Gaussian approximation, we observe that when $(\beta-1)N^{1/2} >> 1$, then

$$K_N(\gamma) \cong \sqrt{\frac{2\pi}{N}}\left(\frac{N}{\gamma}\right)^{N+1} \exp[-n-\gamma] \quad (22)$$

this approximation yields

$$p_0 \cong 0, \quad \bar{n} \cong N\left(1 - \frac{1}{\beta}\right), \quad \sigma_n{}^2 \cong N/\beta. \quad (23)$$

CONCLUSION

In this piece of work we have proposed and examined a non-linear stochastic model for the cooperative behavior. We have obtained probability distribution represented by a random variable n(t), and there from have obtained the expressions for the expected value and variance for $\beta<1$, the process follows geometric distribution, while for $\beta>1$ it follows the quasi-Poissonian distribution.. The presence of two body interaction which render our model non-linear result in to the onset of a cooperative behavior. This leads to macroscopic manifestation of microscopic activities. Therefore, to control the global phenomenon stock market crash, controlling agencies have to upgrade their efforts so that the value of the controlling parameter β does not exceed 1. Furthermore, the stronger the collective behavior due to various economic and psychological reasons and the more homogeneous the investors are, the larger the magnitude of the crash as a manifestation of a social phase transition which is rooted in the deep relationship between statistical mechanics systems and economic systems of interacting agents.

Analysis presented here does not include special assumptions regarding preferences, particular trading strategies, the information process, the origin of the collective behavior effect, etc. Our framework yields a theoretical prediction that fluctuations are expected to peak just before the crash.

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