

## **BEAUTY CONTEST AND LEARNING IN RATIONAL EXPECTATIONS BASED STATIONARY STATE**

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### **ABSTRACT**

*The paper explores the condition of instability of stationary state outcome as predicted by learning literature in defence of Rational Expectations Hypothesis-REH. By using Beauty Contest Parable in Grandmont's temporary equilibrium framework, it is argued that under very weak and reasonable conditions agents extrapolate away from the stationary state. This result is achieved even while ignoring large deviations of the past - a method conclusively criticised by Grandmont.*

**Keywords:** Rational Expectations Hypothesis, Temporary Equilibrium, Beauty Contest Parable, Learning, Stability.

### **INTRODUCTION**

Rational Expectations Hypothesis (REH) has dominated the macroeconomics theory and policy framework since the 1980s (Stiglitz, 2015, 2018). The beginning of the role of expectations as stabilising the system comes with adaptive expectations by Friedman and Phelps (Friedman, 1968). This reversal of role of expectations from Keynes, led to a complete reversal of the desired policy framework - from demand management to monetarism. REH was worked out as a condition on expectations which will ensure a correct forecast about the future values of relevant variables thus ensuring no deviation from full employment in the Arrow Debreu framework. It takes the fundamental problem of asymmetric information in Phelps' island model and offers REH as the solution to it (Phelps et al., 1970). It is noteworthy that the idea of

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individuals' expectations about the system variables being stochastically the same as the system itself first appears in Bachelier's work in the 19th century, and is brought to modern economics by Samuelson, and subsequently Fama, as Efficient Market Hypothesis (Read, 2013). But, for the prices to act as an informationally efficient signal, the necessity of full information about the future markets was made explicit by Lucas, and subsequently accepted by Fama. Lucas concludes that rational individuals optimising with all available information, and the market following a random walk are not sufficient to establish a stock price representing its fundamental value as mean (Lucas, 1978; Read, 2013). Whereas different versions of EMH, representing different availability of information coexisted in finance literature, full information about the future markets and a common knowledge of this was accepted as the first level of abstraction for REH based theories in macroeconomics (Stiglitz, 2015).

The justification of the full information assumption of REH was given by defining the process of 'learning' (Marcel & Sargent, 1989, 1992). This was the second level of abstraction of REH. If individuals can learn the self fulfilling outcome, then the prediction of the model is valid despite the absence of full information about the future. Indeed, it is widely believed that REH based Real Business Cycle theories have certain validity in the economies which are repetitive systems, and possibly provide some scope of learning. Stiglitz called it "agricultural economies" where random shocks are observed (Stiglitz, 2015). Grandmont has shown that the learning processes showing convergence to stationary state outcome (referred as SSO, a dynamic version of self fulfilling equilibrium) is based on deliberately ignoring large deviations from the past by calling it 'shocks' and 'outliers' (Grandmont & Laroque 1990, Grandmont, 1998).

We have used Grandmont's Temporary equilibrium framework and incorporated the beauty contest parable of Keynes in the individual's expectation function to argue that even when 'shocks', 'earthquakes' and 'outliers' are ignored the SSO is an unstable outcome showing that learning is not possible even then. We have shown local instability based on the above-mentioned expectation of only one individual, and certain parametric assumptions.

In section 2, we have explained Grandmont's Temporary equilibrium framework with his general stability and instability results. We have retained the notations of Grandmont 1998, and developed an expectation function in section 3 which incorporates the Keynes's beauty contest parable. This section contains the assumptions and derivations of our result. In section 4 we have discussed the assumptions and their implications.

## **TEMPORARY EQUILIBRIUM FRAMEWORK AND GENERAL STABILITY AND INSTABILITY RESULTS**

Notations and assumptions made in Grandmont's analysis are as follows:

- No strategic considerations among different agents in the market.
- Time is discrete
- State of the system is completely described at every date  $t$  by a single real number  $\mathbf{x}_t$ ,
- $\mathbf{x}_t^e$  denotes average forecast about period  $t+1$  at time  $t$ , each individual's forecast being weighted by their respective relative local contribution to the dynamics of the system.
- The current outcome  $\mathbf{x}_t$  depends on  $\mathbf{x}_t^e$  and  $\mathbf{x}_{t-1}$ , through the temporary equilibrium relation

$$\mathbf{T}(\mathbf{x}_{t-1}, \mathbf{x}_t, \mathbf{x}_t^e) = \mathbf{0} \quad (1)$$

- The analysis will be local, near a stationary state  $\bar{\mathbf{x}}$  defined by

$$\mathbf{T}(\bar{\mathbf{x}}, \bar{\mathbf{x}}, \bar{\mathbf{x}}) = \mathbf{0}$$

$\mathbf{T}$  is supposed to be well defined and continuously differentiable when its arguments are near  $\bar{\mathbf{x}}$ .

The outcome is called equilibrium because it is the result of demand-supply in that period. It is temporary because it is conditioned on the expectation about the next period. Changing expectations will change the demand supply, and current outcome (equilibrium).

- Average expectation function is denoted by

$$x_t^e = \Psi(x_{t-1}, \dots, x_{t-L}) \quad (2)$$

- Assume that either traders know  $\bar{x}$ , or if not, they are prepared to extrapolate constant sequence  $x$  near  $\bar{x}$ . We assume that for all  $\bar{x}$  in the immediate vicinity of  $\bar{x}$ , we have

$$\Psi(x, x, x) = x.$$

- Replacing the forecast in (1) by expression (2), we get the actual temporary equilibrium dynamics:

$$T[x_{t-1}, x_t, \Psi(x_{t-1}, \dots, x_{t-L})] = 0 \quad (3)$$

A discussion of General Instability Result, and General Stability Result is given below in 2.1 (Grandmont, 1998, Grandmont & Laroque, 1990). These results provide a critique of Rational Expectations literature, specifically, learning processes used in it.

### **Grandmont's General Instability and Stability Results**

In the Rational expectation literature, individuals' expectations about the outcome coincide with it; hence the notion of self-fulfilling expectations<sup>i</sup>. This means that individuals have full information about all the markets including all the markets of the future extended to infinitely many periods. Added to it is the assumption that individuals are homogenous who as "signal processors"- a favourite term of Lucas (Fitoussi et al., 1987), process all this information in the same manner, that is through

the same model<sup>ii</sup>. Thus, the same outcome is also the one anticipated by everyone and hence acted upon accordingly. Whereas the assumption of homogenous individuals using the same model to process information has never been adequately justified, the complete informational requirements bestowed upon individuals is totally unrealistic and unacceptable. The assumption of homogenous individuals is never relaxed, and the informational requirements are defended as only the first approximation for the story. The justification of this first approximation is given by bringing in the notion of learning through endogenous expectation functions, and showing convergence to stationary state with the help of it (Maret & Sargent, 1989, 1992). It will be a simplification, but not incorrect to say that these endogenous expectations are essentially learned by taking some kind of average of the past outcome.<sup>iii</sup>

Grandmont provides a comprehensive critique of Rational expectation theory and the associated learning processes, as he contrasts his own framework of Temporary equilibrium to that of Rational expectation. According to Grandmont, the stability of any stationary state outcome in a market where expectations play a crucial role depends upon the range which individuals (traders, for Grandmont) are willing to extrapolate. If "large" deviations have occurred in the past and traders do take those into account while forming expectations about tomorrow, then the stationary state is highly likely to be unstable. Further, if one is talking about a market where the role of expectations is not very important and/or *traders do not take into account the large deviations from the stationary state* while forming their expectations, then the market is likely to be stable. Grandmont argues that in effect, this is precisely what is done by various exponents of Rational expectations theory using assumptions about "Projection facility", which deliberately and quite arbitrarily restrict the range within which traders extrapolate. Any past outcome outside this range must be ignored to ensure stability. According to Grandmont, such restrictions on the range have no basis and showing convergence on the basis of it is devoid of any meaning.

In the General Instability and Stability results, he identifies the conditions under

which the two happen. Conceptually, the sufficient condition for stability is merely negation of sufficient condition for instability, even though mathematically this is not so.

Nevertheless, the point is that it is the 'range of regularities' which determines the stability in the markets where expectations matter. Grandmont argues that the defenders of rational expectation theory ensure the stability of the system by precisely doing this, that is, manipulating the "range of regularities". He gives some examples to show how the restrictions are imposed upon individual's forecast estimators (estimate depends upon the value of past observations) with the help of the some "projection facility" This "projection facility" specifies some lower and upper limit beyond which if estimate falls, it takes the corresponding limit values. The estimator would take values beyond these limits only if large deviations from the past are taken into account. Specifying such "projection facility" from outside the model then means that traders are assumed to ignore such large deviations. The permissible range of deviations (or estimate to be precise) depends on the learning process (i.e. expectation functions) and the structural parameters of the system. Any deviation out of this range (so that true value of estimator falls outside the limits given by projection facility) is termed as "shock" in the rational expectation literature.

According to Grandmont, this method of ensuring stability of actual dynamics with learning is quite arbitrary and cannot be justified in any meaningful way.

## **LEARNING WITH BEAUTY CONTEST PARABLE**

In this section, we use the formulation of Temporary equilibrium dynamics by Grandmont (equations 1,2 and 3 above), while incorporating the idea of strategic behaviour of agents in the sense described in Keynes's Beauty contest parable.

### **Stability of outcome with Beauty Contest Parable**

Following result has been derived to argue that stationary state outcome is unstable,

and even capable of reversing the trends, even though we ignore large deviations lying outside of “projection facility”. In this section, we discuss the impact on the stability of stationary state when the expectations of the individual are such that they depend upon both past observations and the notion about the other individual's expectations (through average expectation in the market), while contemplating that others are also doing the same, hence the need to consider that also while forming one's expectations, and so on. The model below is a mathematical representation of Keynes's beauty contest parable.

In the next sections, the results and the interpretation of underlying assumptions are discussed. The notations used by Grandmont are retained for our result.

### **A case of instability in the absence of “shocks” when individuals are strategic**

In this section we explore the stability of SSO when the 'shocks' are ignored. We have assumed that agents in the market are strategic in the sense described in the Keynes's famous beauty contest parable to describe the behaviour in financial market (Keynes 1936). Agents will extrapolate to predict future outcome by contemplating the expectations of other agents. Further she assumes other agents to be doing the same, and everyone assumes that everyone assumes others to be doing it, and so on. We specify a model which allows us to analyse the deviation around any stationary state outcome (SSO). From the point of view of the stability of SSO in response to random disturbances, the impacts of two different types of individuals are different. The first type predicts the outcome to change by less than aggregated or average expectation, and assumes others to be predicting the same, and further assumes that others also assume the same, and so on. The second type predicts it to be changing more, and so on. Whereas the first type causes the outcome to gravitate towards the stationary state, the second causes the deviation to amplify in one direction.

We specify the expectation function of any individual  $i$  as follows:

$$\begin{aligned} x^{e_i} = & \psi_i[x_t, \dots, x_{t-j}; f_i(\sum_{l_1} \psi_{il_1}(x_t, \dots, x_{t-j}; f_{il_1}(\sum_{l_2} \psi_{il_1l_2}(x_t, \dots, x_{t-j}; f_{il_1l_2}(\sum_{l_3} \psi_{il_1l_2l_3}(x_t, \dots, \\ & + \dots, x_{t-j}); f_{il_1l_2l_3}(\dots))\dots)] \end{aligned}$$

Where  $j = 0, 1, 2, \dots, L$

$$l_1, l_2, l_3, \dots, l_n = 1, 2, 3, \dots, n$$

$n$  is the number of individuals.

$x^{e_i}$  is  $i$ 's expectation about period  $t+l$  at period  $t$ .

$Y_i$  is  $i$ 's expectation function.

$f$  denotes the market aggregation of an individual's expectation function (i.e.,  $x^e = f(Y_1, Y_2, \dots, Y_n)$ ).

$f_i$  is individual  $i$ 's notion of  $f$ .

$Y_{il_1}, Y_{il_2}$  denote  $i$ 's notion of  $l_1$ 's expectation function,  $i$ 's notion of  $l_1$ 's notion of  $l_2$ 's expectation function and so on.

$f_{il_1}, f_{il_2}, \dots$  are interpreted similarly.

Stationary state: Let  $\bar{x}$  be the stationary state. If  $x_t = x_{t-1} = \dots = x_{t-L} = \bar{x}$ , then  $x^{ei} = \bar{x}$ .

All the partial derivatives below are evaluated at the stationary state.

$$c_{ij} = \frac{\delta \psi_i}{\delta x_{t-j}}, c_{il_1j} = \frac{\delta \psi_{il_1}}{\delta x_{t-j}}, \text{ and so on.}$$

$$d_i = \frac{\delta \psi_i}{\delta f_i}, d_{il_1} = \frac{\delta \psi_{il_1}}{\delta f_{il_1}}, \text{ and so on.}$$

$$\alpha_{il_1} = \frac{\delta f_i}{\delta \psi_{il_1}}, \alpha_{il_1l_2} = \frac{\delta f_{il_1}}{\delta \psi_{il_1l_2}}, \text{ and so on.}$$

We make the following assumption:

$$\alpha_{il_1} = \alpha_{il_2} = \alpha_{il_1 l_2 l_3} = \alpha = \frac{1}{n} \quad (\text{A1})$$

$$d_{il_1}, d_{il_2}, d_{il_1 l_2 l_3}, \dots \geq 1 \quad (\text{A2})$$

$$\sum_{l_T} \alpha c_{il_1 \dots l_T} dx_{t-j} = c_{ij} dx_{t-j}, \text{ where } T = 1, 2, 3, \dots \quad (\text{A3})$$

Now, linearizing equation (1), around the stationary state, we get:

$$\begin{aligned} dx^0 &= \sum_j c_{ij} dx_{t-j} + d_i [\alpha \sum_{l_1} \sum_j c_{il_1 j} dx_{t-j} + \alpha^2 \sum_{l_1} \sum_{l_2} d_{il_1} c_{il_1 l_2 j} dx_{t-j} + \\ &\quad \alpha^3 \sum_{l_1} \sum_{l_2} \sum_{l_3} \sum_j d_{il_1} d_{il_2} c_{il_1 l_2 l_3 j} dx_{t-j} + \dots] \end{aligned} \quad (4)$$

We fix  $j = J$  ( $J$  is any of the numbers  $1, 2, \dots, L$ ) and analyse the term in the bracket in the above equation. We have:

$$\begin{aligned} &\alpha \sum_{l_1} c_{il_1 j} dx_{t-j} + \alpha^2 \sum_{l_1} \sum_{l_2} d_{il_1} c_{il_1 l_2 j} dx_{t-j} + \dots \\ &\quad + \alpha^T \sum_{l_1} \dots \sum_{l_T} d_{il_1} d_{il_2} \dots d_{il_1 \dots l_{T-1}} c_{il_1 \dots l_{T-1} j} dx_{t-j} \end{aligned} \quad (5)$$

The equation above shows the first, second, and  $T^{\text{th}}$  terms. Similarly, the  $(T+1)^{\text{th}}$  term in (5) is:

$$\alpha^{T+1} \sum_{l_1} \dots \sum_{l_{T+1}} d_{il_1} d_{il_2} \dots d_{il_1 \dots l_{T+1}} c_{il_1 \dots l_{T+1} j} dx_{t-j}$$

For  $T \neq n$ :

$$\sum_{l_T} \alpha c_{il_1 \dots l_T j} dx_{t-j} = \sum_{l_T} \alpha c_{il_1 \dots l_{\rho} \dots l_T j} dx_{t-j}$$

Where  $\rho = 1, 2, \dots, n$

$$\begin{aligned} &\rightarrow n \sum_{l_T} \alpha c_{il_1 \dots l_T j} dx_{t-j} = \sum_{l_T} \sum_{\rho} \alpha c_{il_1 \dots l_{\rho} \dots l_T j} dx_{t-j} \\ &\rightarrow \sum_{l_T} \alpha c_{il_1 \dots l_T j} dx_{t-j} = \alpha \sum_{l_T} \sum_{\rho} \alpha c_{il_1 \dots l_{\rho} \dots l_T j} dx_{t-j} \\ &\rightarrow d_{il_1} d_{il_2} \dots d_{il_1 \dots l_{T-1}} \sum_{l_T} \alpha c_{il_1 \dots l_T j} dx_{t-j} \leq \alpha d_{il_1} d_{il_2} \dots d_{il_1 \dots l_T} c_{il_1 \dots l_{\rho} \dots l_T j} dx_{t-j} \quad [\text{due to (A2)}] \\ &\rightarrow \sum_{l_1} \dots \sum_{l_T} d_{il_1} d_{il_2} \dots d_{il_1 \dots l_{T-1}} \sum_{l_T} \alpha c_{il_1 \dots l_T j} dx_{t-j} \leq \end{aligned}$$

$$\begin{aligned}
& \alpha \sum_{l_1} \dots \sum_{l_T} d_{i_l} d_{i_l l_2} \dots d_{i_l \dots l_T} \sum_{l_T} \sum_{\rho} \alpha c_{i_l \dots l_{\rho} l_{\rho} \dots l_T} dx_{l-T} \\
\rightarrow & \alpha^T \sum_{l_1} \dots \sum_{l_T} d_{i_l} d_{i_l l_2} \dots d_{i_l \dots l_{T-1}} \sum_{l_T} \alpha c_{i_l \dots l_{\rho} l_{\rho} \dots l_T} dx_{l-T} \\
\leq & \alpha^{T+1} \sum_{l_1} \dots \sum_{l_T} d_{i_l} d_{i_l l_2} \dots d_{i_l \dots l_T} \sum_{l_T} \sum_{\rho} \alpha c_{i_l \dots l_{\rho} l_{\rho} \dots l_T} dx_{l-T}
\end{aligned} \tag{6}$$

Consider the right-hand side of equation (6). Due to (A3), this is equal to:

$$\begin{aligned}
& \sum_{l_1} \dots \sum_{l_T} d_{i_l} d_{i_l l_2} \dots d_{i_l \dots l_T} \sum_{l_T} \sum_{\rho} \alpha c_{i_l \dots l_{\rho} l_{\rho} \dots l_T} dx_{l-T} \\
= & \sum_{l_1} \dots \sum_{l_T} d_{i_l} d_{i_l l_2} \dots d_{i_l \dots l_T} \sum_{l_{T+1}} \alpha c_{i_l \dots l_{T+1}} dx_{l-T}
\end{aligned}$$

But this is nothing but the  $(T+1)^{\text{th}}$  term. The left-hand side of equation (6) is the  $T^{\text{th}}$  term. So, the  $(T+1)^{\text{th}}$  term is at least as large as the  $T^{\text{th}}$  term. Therefore, equation (5) which is the sum of an infinite series, after  $(n-1)^{\text{th}}$  term, is non-declining. Hence, the sum i.e.,  $i$ 's expectation tends to infinity.

Now,  $x^e = f(\psi_1, \dots, \psi_n)$

$$\rightarrow dx^e = \sum_k \alpha_k d\psi_k, \quad k = 1, 2, \dots, n$$

Since,  $\alpha_i = \alpha$  is positive and finite, and  $\psi_i$  tends to infinity,  $dx^e$  would also tend to infinity.

***So, the system becomes locally unstable (and hence globally unstable also) regardless of the actual learning process of other individuals.***

Therefore, with only one individual's expectation function of the above-mentioned form and satisfying assumptions (A1), (A2), and (A3), the market is unstable.

This result, its meaning and implications are discussed in the following sections.

### **Explaining the above expectation function of the strategic agent**

As mentioned in the previous section, the expectation function used by us is based on Keynes's beauty contest parable. While forming one's expectation about tomorrow, individuals act strategically taking into account the average expectation. Such

behaviour has been indicated by Keynes by drawing a parallel with the beauty contest. In chapter 12 of The General theory Keynes indicates the agents' behaviour when all they are interested in is the short-term variations in the asset prices. In this venture they want to match their expectation (forecast) with the average expectation. They react as in, ".....a game of snap, of Old Maid, of Musical Chair - a past time in which he is victor who says snap neither too soon nor too late, who passes the Old Maid to his neighbour before the game is over, who secures the chair for himself when the music stops. Or, to change the metaphor slightly, professional investment may be linked to those newspaper competitions in which the actors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole, so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgement, are really the prettiest, nor even those which the average opinion genuinely thinks the prettiest. We have reached a third degree where we devote our intelligence to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees" (Keynes, 1936).

As it is clear from the above quotation, Keynes expected his readers to be surprised by the fact that some people are known to do this exercise upto fourth or fifth level.

But it is assumed in the previous section that individuals do this exercise infinitely many times. This enables one to have an endogenous expectation function as ultimately the expectation would depend upon the past observations.

Keynes own emphasis upon the importance of convention in the expectation formation (endogeneous expectation function, mathematically speaking) as is clear from this statement of Keynes, "We are assuming, in effect, the existing market valuation, however, arrived at, is uniquely correct in relation to our existing knowledge of the facts which will influence the yield of the investment, and that it will only change in

proportion to changes in this knowledge, though philosophically speaking, it cannot be uniquely correct, Nevertheless, the above conventional method of calculation will be compatible with a considerable measure of continuity and stability in our affairs, so long as we can rely on the maintenance of the convention" (Keynes, 1936).

The emphasis on convention, modified by incorporating the changes is further made explicit in the later notes of Keynes (Keynes, 1937).

## INTERPRETING THE ASSUMPTIONS AND THE RESULT

### Interpretation of Assumption 2:

Now we take up the assumption 2 that we've made. This assumption states that there is an individual  $i$  such that all the partial derivatives  $ds_{...}$  are as large as one. This means that if there is a small change in  $i$ 's notional average expectation then his expectation changes by more than the former. Also,  $i$  thinks that all other individuals react similarly to any small change in their respective notional average expectations. Further,  $i$  thinks that everyone thinks similarly about everyone else.

To clarify the point further we assume a hypothetical share market where infinitesimal changes occur. The assumption says that if  $i$  thinks that market is bullish about any share (i.e., average expectation about the share is that its Price would go up), then because of this, her expectation about that share price would be even higher than the average expectation. That is, she is more bullish than the market. Further,  $i$  thinks that everyone wants to be more bullish than the market, i.e., everyone increases one's expected increase in price by more than the increase in average expected increase in price. Also,  $i$  thinks that this tendency to be more bullish than the market for everyone is a common belief in the market. Common belief is used here to mean the similar thing as "common knowledge", except that it's about the notion or belief of individuals rather than any actual fact.

The question remains that why is it that the individual is more bullish than the market about that particular share. Why is that she wants to outperform others in getting the

share which she thinks that everyone is more bullish about than the market and is trying to outperform others<sup>iv</sup>. This is possible only if i thinks that due to any increase of average expectations over the stationary state, the actual outcome would increase more than average expectation. Also, i is thinking that everyone is thinking in the same manner and this is a common belief.

### **Interpretation of Assumption 3:**

To get the result, assumption 3 is used. *Assumption 3 implies that i thinks that an average person reacts to (t-j)<sup>th</sup> period derivation as much as she herself reacts. Further, i thinks that everyone thinks on an average everyone else reacts to (t-j)<sup>th</sup> period deviation by the same magnitude, and so on.*

This means that i has certain notion of people on an average reacting to (t-j)<sup>th</sup> period deviation and this she takes to be the "common notion", and then, she reacts only that much.

This assumption could be justified as i observed only the outcome and then forms some notion of average reaction to periods deviation, which is in accordance to her notion of other learning parameters of everyone, the structural parameters of the market and the past outcomes. Indeed, if one makes a vector of everyone's learning parameters and one's notion of structural parameters of market, then there can be possibly a number of values of vector which explains the past outcomes. Assumption states that for the individual i all the individuals use the same vector as her.

Essentially, individual thinks her opinion to be representative of the population.

The requirement is that i chooses that one which satisfies (A3) and uses it to make her forecast.

### **Interpreting Assumption 1:**

Assumption 1 says that all individuals have equal weightage in the market. This is a

very unrealistic assumption. The defence of this assumption can be given by taking any individual whose weight is  $M$  times the weight of the smallest individual, as  $M$  individuals all having the same expectation function as the large (parent) individual. In that case  $N$  would represent in the population such constructed, rather than the number of individuals in the actual population.

Clearly, Assumption 1 is a simplifying assumption.

## Results

The agents for whom the above assumptions are satisfied are the source of instability, as they drive the market in one direction in response to any random change. These are the bullish agents who want to outperform the market. The same set of agents are also bearish when there is a decline in price of any asset, as again, they want to sell before everyone else does, bringing the market to crash. The result in section 3.2 describes the deviation and explosive divergence from SSO if sufficient numbers of agents satisfy the assumptions.

## CONCLUSION

We have shown that no learning of SSO is possible even if we ignore the large deviations when we allow the agents to become strategic in the sense of beauty contest. While the insistence on ignoring the large deviations was never reasonable, and yet necessary as shown by Grandmont, the fact that even that may not be sufficient for stability and learning is a stronger rejection of the REH-learning literature.

<sup>i</sup>To be precise, REH insists only on the same stochastic distribution of the individual's expectations and the actual outcome. We are simply following Grandmont's formulation here.

<sup>ii</sup>Signal processing individuals try to differentiate between the noise, and the real changes.

<sup>iii</sup>This approach is very different from the notion of Bayesian learning, in which subjective probabilities of individuals is the starting point(Feldman, 1987).

<sup>iv</sup>In chapter 12, The General Theory, Keynes talks of such behaviour when people have poor belief about some asset. He says, "The actual, private object of the most skilled investment to-day is to beat the gun", as Americans so well express it, to outwit the crowd, and to pass the bad, or depreciating, half-crown to other fellows."

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