SHRI RAM COLLEGE OF COMMERCE

BA (HONS.) ECONOMICS – SEM 4C SESSION 2019-2020

INTERMEDIATE MICROECONOMICS -2

Game Theory – Definitions -1

Note: *Put these definitions to work with the help of examples done in class and from the readings. You may also create your own examples in order to better grasp the definitions. This is rigorous work and should be taken seriously.*

- A strategy is a complete contingent plan of action.
- A game in *strategic* form or *normal* form is a triple $\Gamma \equiv (N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$ in which:
  - **N** $= \{1, 2, ..., n\}$ is a finite set of players
  - $\mathbf{S_i}$ is the set of strategies of player i, for every player $i \in N$ - the set of *strategy profiles* is denoted as $S \equiv S_1 \times ... \times S_n$,
  - $\mathbf{u_i} : S \rightarrow R$ is a utility function that associates with each profile of strategies $s \equiv (s_1, ..., s_n)$, a payoff $u_i(s)$ for every player $i \in N$.

  When the S_i is finite *for each* $i \in N$, we will refer to Γ as a finite game.

  A strategy profile of all the players will be denoted as $s \equiv (s_1, ..., s_n) \in S$. A strategy profile of all the players excluding a Player i will be denoted by s_{-i}. The set of all strategy profiles of players other than a Player i will be denoted by S_{-i}.

- A strategy s_i of player i ***strictly dominates*** her strategy s'_i if for all $s_{-i} \in S_{-i}$,
$$u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$$

- A strategy $s_i \in S_i$ for Player i is ***strictly dominant*** if it strictly dominates every $s'_i \in S_i \setminus \{s_i\}$.

- A strategy si ***weakly dominates*** strategy s'_i if for every $s_{-i} \in S_{-i}$,
$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}),$$
  with strict inequality holding for some s_{-i}.

- A strategy s_i is ***weakly dominant*** if it weakly dominates every other strategy $s'_i \in S_i \setminus \{s_i\}$.

- A strategy $s_i \in S_i$ for Player i is ***strictly dominated*** if there exists $s'_i \in S_i$ such that s'_i strictly dominates s_i, i.e., *for every* $s_{-i} \in S_{-i}$, we have
$$u_i(s_i, s_{-i}) < u_i(s'_i, s_{-i}).$$

- A strategy $s_i \in S_i$ for Player i is ***weakly dominated*** if there exists $s'_i \in S_i$ such that s'_i weakly dominates s_i, i.e., *for every* $s_{-i} \in S_{-i}$, we have
$$u_i(s_i, s_{-i}) \leq u_i(s'_i, s_{-i})$$
  with strict inequality holding for some s_{-i}.

- A strategy profile $(s^*_1, ..., s^*_n)$ in a strategic form game $\Gamma \equiv (N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N})$ is a **Nash equilibrium** of Γ if *for all $i \in N$*
$$u_i(s^*_i, s^*_{-i}) \geq u_i(s_i, s^*_{-i}) \ \forall s_i \in S_i.$$
- A strategy profile $(s^*_1, ..., s^*_n)$ in a strategic form game $\Gamma \equiv (N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N})$ is a **strict Nash equilibrium** of Γ if *for all $i \in N$*
$$u_i(s^*_i, s^*_{-i}) > u_i(s_i, s^*_{-i}) \ \forall s_i \in S_i \setminus \{s^*_i\}.$$

- A strictly dominated strategy will never be part of a Nash Equilibrium. A weakly dominated strategy may be part of a Nash Equilibrium. A weakly dominated strategy will never be part of a strict Nash Equilibrium.
- A strategy s_i of Player i is a ***best response*** to the strategy s_{-i} of other players -i if
$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}) \ \forall s'_i \in S_i.$$
The set of all best response strategies of Player i given the strategies of other players, s_{-i}, is denoted by
$$B_i(s_{-i}) := \{s_i \in S_i : u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}) \ \forall s'_i \in S_i\}.$$

  Now, a strategy profile $(s^*_1, ..., s^*_n)$ is a Nash equilibrium if for all $i \in N$,
$$s^*_i \in B_i(s^*_{-i}).$$
Hence, Nash equilibrium requires non-emptiness of best response set at the equilibrium strategy profile.
s^* is a strict Nash equilibrium *iff* $\forall\, i \in N$, $s^*_i \in B_i(s^*_{-i})$ and $B_i(s^*_{-i})$ is a singleton set, i.e., $\{s^*_i\} = B_i(s^*_{-i})$.

- If s^*_i is a strictly dominant strategy of Player i, then $\{s^*_i\} = B_i(s_{-i})$ for all $s_{-i} \in S_{-i}$. Hence, if $(s^*_1, ..., s^*_n)$ is *a strictly dominant strategy profile*, it is a unique Nash equilibrium. If s^*_i is a weakly dominant strategy of Player i, then $s^*_i \in B_i(s_{-i})$ for all $s_{-i} \in S_{-i}$. Hence, if $(s^*_1, ..., s^*_n)$ is *a weakly dominant strategy profile*, it is *a* Nash equilibrium.
- A two-player normal form game is ***symmetric*** if the players' sets of strategies are the same and the players' payoffs are represented by the payoff functions u_1 and u_2 for which $u_1(s_1, s_2) = u_2(s_2, s_1)$ for every pair (s_1, s_2). *E.g., The Prisoners' Dilemma.*
- A strategy profile s^* in a normal form game in which each player has the same set of strategies is a ***symmetric Nash Equilibrium*** if it is a Nash Equilibrium and s_i^* is the same for every player i. (*Check if the Prisoners' Dilemma and The Battle of sexes are games with symmetric Nash equilibria.)*