Cournot duopoly : Perfect cartel

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The Nash equilibrium of the Cournot duopoly example discussed before is:

$$(q_1^*, q_2^*) = (\frac{\alpha - c}{3}, \frac{\alpha - c}{3})$$

Putting these equilibrium values in the profit functions of the firms, we get the following equilibrium profits:

$$\Pi_1^* = \Pi_2^* = \frac{(\alpha - c)^2}{9}$$

Perfect Cartel

Firms choose their outputs to maximize the joint profit function

$$\Pi = \Pi_1 + \Pi_2 = \begin{cases} (\alpha - c - q_1 - q_2)q_1 + (\alpha - c - q_1 - q_2)q_2 & \text{if } q_1 + q_2 \le \alpha \\ -cq_1 - cq_2 & \text{if } q_1 + q_2 > \alpha. \end{cases} \tag{1}$$

The first order conditions are

$$\frac{\partial(\Pi_1 + \Pi_1)}{\partial q_1} = \frac{\partial(\Pi_1 + \Pi_1)}{\partial q_2} = \alpha - c - 2q_1 - 2q_2 = 0$$

Since the firms produce identical products and face the same constant marginal cost c, they would produce the same quantities:

$$q_1^* = q_1^* = q^*$$

This gives us:

$$q^* = \frac{\alpha - c}{4}$$

Putting this in the joint profit function, we get:

$$\Pi^* = \frac{(\alpha - c)^2}{4}$$

Dividing this joint profit equally among the firms, we get:

$$(\Pi_1^*, \Pi_2^*) = (\frac{(\alpha - c)^2}{8}, \frac{(\alpha - c)^2}{8})$$

Note that each firm's profit is more in the perfect cartel outcome than in the Nash equilibrium outcome. Hence, the Nash equilibrium is *pareto dominated* by firms choosing quantities to maximize joint profits.