classmate
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Problem set on Unconstraint optimization

Q-1 Consider the monopolist producing two distinct goods. The cost function is given by

$$TC(q_1, q_2) = q_1 + kq_2$$

where the constant $k \in (0,1)$.
Demand functions are given by

$$q_1(P_1, P_2) = q_2(P_1, P_2) = (P_1, P_2)^{-3}$$

where $P_1 > 0$, $P_2 > 0$

Find the optimal prices for the monopolist. For what values of k one of the products is priced under marginal cost?

Q-2 Consider the following maximization problem

$$\max_{x,y} f(x, y, a, b) = ax^2 - x + by^2 - y$$

(a) Find all the stationary pts of the function.
(b) Under what conditions for a and b is the stationary pt(s) found in (a) is a maximum
(c) Use the envelope theorem to find out how the value fn $f(x^*(a,b), y^*(a,b), a, b)$ varies as a varies. Check the result with the general method too.
(d) Under what conditions on a and b is the function f concave in x and y? when it is convex in x and y?

Q-3 Show that the function $f(x,y,z,w) = -w^2 + 2wx - x^2 - y^2 + 4yz - z^2$ is not concave.

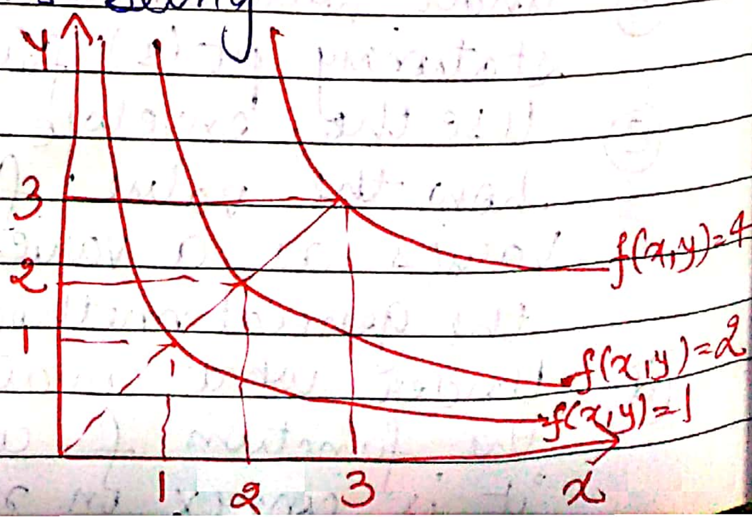
Q-4 The function C of many variables and the function D of a single variable are both convex. Define the function f by $f(x,k) = C(X) + D(k)$. Show that f is a convex function (without assuming that C and D are differentiable)

Q-5 The functions f and g of a single variable are concave (but not necessarily differentiable). Is the function h defined by $h(x) = f(x) \cdot g(x)$ necessarily quasiconcave?

Q-6 The Three curves in the figure below are the sets of points for which the value of the function f (of two variables x and y) is equal to 1, 2, and 4. Are these curves consistent or inconsistent with the function's being

(a) quasiconcave?

(b) Concave?



Q-7 Solve the problem

$$\max_{x,y} (\min) \quad 3 + x^3 - x^2 - y^2$$

Subject to $x^2 + y^2 \leq 1$ and $x \geq 0$

Q-8 Find all the local maxima (if any) of the following functions. For each local maxima that you find, determine, if possible, whether it is a global maximum.

(a) $f(x,y) = \frac{1}{3}x^3 + 2xy - 2y^2 - 6x$

(b) $f(x,y) = 3xy - x^3 - y^3$

Q-9 Solve the problem

$$\max_{x} \int_0^1 -(x-y)^2 g(y)\,dy$$

where g is a function for which ~~$0 \leq g(y) \leq 1$~~

$0 \leq g(y) \leq 1$ for all y

and $\int_0^1 g(y)\,dy = 1$ [Hint apply Leibniz's formula]