

Mathematics of Finance

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B.Com. (Hons.) IV Semester

Section A

Business Mathematics



Session Details

This Session Covers

- Effective and Normal Rate of Interest
- Present value of Amount
- Equation of Value
- Depreciation

Solved illustrations and Practice Handouts are provided separately



Nominal and Effective Rate of Interest

In this topic we will firstly discuss about the nominal rate of interest then we will discuss about the effective rate of interest meaning, formula and examples to compute effective rate of interest.

The nominal rate of interest is the actual rate of interest which is stated on the any investment or loan.

When interest is compounded, more than one year, then the actual interest rate p.a. is lesser than the effective rate of interest.



The effective rate of interest is the equivalent annual rate of interest which is compounded annually. Further, the compounding must happen more than once every year.



Relationship between Nominal & Effective Rates

- Let r = Nominal Rate of Interest p.a.
- m = No. of conversion periods during a year
- P = Principal Amount
- r_e = Effective Rate p.a.
- After a year, $A = P \left(1 + \frac{r}{m}\right)^m$ as well as $A = P(1 + r_e)$
- $P(1 + r_e) = P \left(1 + \frac{r}{m}\right)^m$ which also mean $r_e = \left(1 + \frac{r}{m}\right)^m - 1$



Force of Interest

- Force of Interest = Nominal rate of interest that is compounded continuously to result in an effective rate
- That is if m tends to infinity, r_e is force of interest $\lim_{m \rightarrow \infty} r_e$

$$r_e = \lim_{m \rightarrow \infty} \left[\left(1 + \frac{r}{m} \right)^m - 1 \right] = \lim_{m \rightarrow \infty} \left(1 + \frac{r}{m} \right)^m - 1$$

$$r_e = \lim_{m \rightarrow \infty} \left[\left(1 + \frac{r}{m} \right)^{\frac{m}{r}} \right]^r - 1 = \left[\lim_{m \rightarrow \infty} \left(1 + \frac{r}{m} \right)^{\frac{m}{r}} \right]^r - 1$$

Let $x=r/m$, then as $m \rightarrow \infty$ implies that $x \rightarrow 0$

$$r_e = \left[\lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} \right]^r - 1 = e^r - 1 \text{ as } \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = e$$
$$r_e = e^r - 1$$



Illustrative Problems

- Find the effective rate equivalent to the nominal rate 6% converted (i) monthly, (ii) continuously.
- Mr X took a loan of ₹ 2000 for 6 months. Lender deducts ₹ 200 as interest while lending. Find the effective rate of interest charged by lender.

Solⁿ:-

$$(i) \quad Y_e = \left(1 + \frac{r}{m}\right)^m - 1$$

$$r = 0.06 \quad m = 12$$

$$Y_e = \left(1 + \frac{.06}{12}\right)^{12} - 1$$

$$Y_e = 1.0616 - 1 = 0.0616$$

$$Y_e = 0.0616 \text{ (OR) } 6.16\%$$

(ii)

$$Y_e = e^r - 1$$

$$= e^{.06} - 1$$



taking the value of Table V from the back side of the Book

$$Y_e = 1.0618 - 1$$

$$Y_e = 0.0618 \text{ (OR) } 6.18\%$$

Solⁿ:-

Interest deducted by money lender is ₹ 200
₹ 200 is the interest on ₹ 1800.

$$\therefore \text{Rate is } \frac{200}{1800} = \frac{1}{9} \quad m = 2 \text{ (semi annual)}$$

$$Y_e = (1+i)^m - 1 \Rightarrow \left(1 + \frac{1}{9}\right)^2 - 1$$

$$= 0.23456$$

$$= 23.45\% \text{ (OR)}$$



Present Value of Amount

The Present Value is an amount that need to be invested at present so that a specified is received after a specified time with a given rate of interest.

- The compound amount formula $A = P(1 + i)^n$
- Divided both sides by $(1 + i)^n$
- $P = A(1 + i)^{-n}$



Present Value of Amount at continuous compounding

The compound amount formula $A = Pe^{rt}$

- Divided both sides by e^{rt}

$$P = Ae^{-rt}$$



Illustrative Problems

- Mr. X left ₹50000 to be divided between his two daughters A and B. A's share was to amount to a certain sum of money at the end of 5 years and B's share was to amount to an equal amount at the end of 7 years. If the rate of interest is 6 % compounded annually, find the amount.

Let A's share amount to £A at the end of 5 years.
Then B's share also amounts to £A at the end
of 7 years.

Rate of interest is 6% compounded annually.

$$\therefore \text{P.V of A's amount} = A(1+0.06)^{-5}$$

$$\text{P.V of B's amount} = A(1+0.06)^{-7}$$

$$A(1+0.06)^{-5} + A(1+0.06)^{-7} = 50,000$$

$$A \left[(1.06)^{-5} + (1.06)^{-7} \right] = 50,000$$

taking the value from the P.V of £1
table

$$A [0.7472 + 0.6650] = 50,000$$

$$A [1.4122] = 50,000$$

$$A = \frac{50,000}{1.4122} = 35,405.75$$



Equation of Value

An equation of value is the equation which state that the sum of value on a given date for **one set of obligation** is **same as**, the sum of the value on the same date, of **another set of obligation**



We can also describe equation of value as...

Amount of
loan taken



Amount of
loan paid

At a particular
time period



Focal Date

For comparing the value of two sets of obligation we need to select a date, that date is considered as focal date.

We can select any date as a focal date for comparison purpose



Illustrative Problems

A debt of ₹5000 due 5 years from now and ₹5000 due 10 years from now is to be repaid by a payment of ₹2000 in 2 years, a payment of ₹4000 in 4 years and final payment at the end of 6 years. If the interest rate is 7% compounded annually, how much is final payment?

A loan of ₹30,000 due 6 years from now is instead to be paid off by three payments: ₹5000 from now, ₹15,000 in three years and a final payment of Rs. ₹4750 at the end of n years. If the rate of interest is 6% compounded annually, find the value of n .

let x be the final payment.
We can select any date as focal date for comparison purpose.

let the focal date is 6 years from now.

value of old obligation at 6 yrs. from now is
 $5000(1.07)^1 + 5000(1.07)^{-4}$ Debt

value of new obligation at 6 yrs from now is
 $2000(1.07)^4 + 4000(1.07)^2 + x$ - payment

eqⁿ of value is

$$2000(1.07)^4 + 4000(1.07)^2 + x = 5000(1.07) + 5000(1.07)^{-4}$$

checking these
value from F.V
& P.V Tables of
₹ 1

$$2000(1.3107) + 4000(1.1449) + x + 5000(1.07) + 5000(1.07)^{-4}$$

$$2621.4 + 4579.6 + x = 5350 + 3814$$

$$7201 + x = 9164$$

$$x = ₹ 1963$$

Sol.
Let the P.V. of the loan (due) must be equal to the P.V. of the three payments

$$30000(1+0.06)^{-6} = 5000 + 15000(1+0.06)^{-3} + \frac{4750x_n}{(1+0.06)^n}$$

$$30000(.70496) = 5000 + 15000(.839619) + 4750(1.06)^{-n}$$

$$21148.8 = 5000 + 12594.285 + 4750(1.06)^{-n}$$

$$4750(1.06)^{-n} = 3554.515$$

$$(1.06)^{-n} = \frac{3554.515}{4750} = 0.7483189$$

$$-n \log(1.06) = \log(0.7483189)$$

$$-0.025305865n = -0.1259$$

$$n = 4.9751$$

∴ the final payment should be made at the end of 5 years (approx.) from now.



Meaning of Depreciation

Depreciation is an accounting procedure for allocating the cost of the capital or non-current assets such as buildings , vehicles , machinery tools over their useful life. It is important to note the depreciation amount are estimate .

Depreciation expenses will allow firms to recapture the original amount of assets indeed to recover the original investment.

Depreciation can also be viewed as a decline in the value of assets due to age , wear and tear , or decreasing efficiency. All the assets , depreciate in value as they the get older.



Depreciation Related terms:

Original Cost/cost:

- The original cost of an asset is the **amount** of money paid for an asset which includes sales tax , delivery charges , installation charges

Useful life of Asset:

- Useful life is the life expectancy of the assets or the number of years the asset is expected to be used

Salvage Value:

- Salvage Value which is also Known as scrap value or trade-in value of the asset at the end of its useful life



Methods of Depreciation

**Straight Line Method
(SLM)**

**Written Down Method
(WDV)**



Straight Line Method (SLM)

This straight line method is very simple and the most common method. The amount of Depreciation is spread evenly to each year throughout the useful life of the asset.

The formulae for finding the annual Depreciation, annual rate of Depreciation and Book value are given as follows:

- Total Depreciation = $C - S$
- Annual Amount of Dep. = $(C - S)/n$
- Annual Rate of Depreciation = Annual depreciation/Total Depreciation
- Book Value at the end of k^{th} year = $C - K(\text{Annual Depreciation})$



Written Down Method (WDV)

WDV is an accelerated method of depreciation in which higher depreciation charge is deducted in the early life of the assets and becomes smaller in the later years.

The formulae associated with this method for finding the annual rate of Depreciation and Book value and Depreciation at k^{th} period are given as follows:

- Book Value at the end of k^{th} year = $c(1 - r)^k$
- Annual Rate of Depreciation, $r = 1 - \sqrt[n]{\frac{S}{C}}$
- Depreciation at k^{th} period = $r * BV_{k-1}$



Illustrative Problems

1. A computer whose cost is 10,00,000 will depreciate to a scrap value of 1,00,000 in 5 years. What is the book value of computer at the end of 4th year?

- If the reducing balance method of depreciation is used.
- If the straight-line method of depreciation is used.

2. An asset costing ₹4500 will depreciate to a scrap value of ₹500 in 10 years. Find the rate of depreciate.

Solⁿ:

Let the depreciation rate be r .
Then

$$C(1-r)^5 = 1,00,000$$

$$10,00,000(1-r)^5 = 1,00,000$$

$$(1-r)^5 = 1/10$$

taking log both the side

$$5 \log(1-r) = \log 1 - \log 10 \Rightarrow 0 - 1$$

$$\log(1-r) = \frac{-1}{5} = -0.2 = -0.2 + 1 - 1 = \bar{1}.8$$

$$(1-r) = \text{AL}(\bar{1}.8) = 0.631$$

The Book value ~~after 5~~ at the end of 4th year

$$= 10,00,000(1-r)^4$$

$$= 10,00,000 \times (.631)^4$$

$$= ₹1,58,532.182$$

(ii) As per SLM, the annual Depreciation

$$= \frac{C - S}{\text{useful life}} = \frac{10,00,000 - 1,00,000}{5}$$

$$= \frac{9,00,000}{5} = ₹1,80,000$$

The Book value at the end of 4th year

$$= 10,00,000 - 4(1,80,000)$$

$$= ₹2,80,000$$

Solⁿ:-

Let r be the rate of Depreciation.

$$C(1-r)^n = 500$$

$$4500(1-r)^{10} = 500$$

$$(1-r)^{10} = \frac{500}{4500} = \frac{1}{9}$$

taking log both the side

$$10 \log(1-r) = \log(1/9)$$

$$= \log 1 - \log 9$$

$$10 \log(1-r) = 0 - .9542$$

$$\log(1-r) = -.09542 \div 10$$

$$\log(1-r) = \bar{7}.9046$$

$$(1-r) = AN(\bar{7}.9046)$$

$$(1-r) = 0.8028$$

$$r = 1 - 0.8028$$

$$= .1972 \text{ or } .1972 \times 100$$

$$= 19.72\%$$

∴ the depreciation rate is 19.72%.

Questions for Mathematics of Finance

Ques:1. (i) A certain sum of money is invested at 4% compounded annually. The interest for second year is ₹ 25. Find the interest for 3rd year.

(ii) A sum of money is put at compound interest for two years at 20% p.a. It would fetch ₹482 more, if the interest were payable half yearly than if it were payable yearly. Find the sum.

Ques:2. A sum of money is deposited in a bank which compound interest semiannually. The amount at the end of 4 years is ₹6333.85 and the amount became ₹8023.53 at the end of 8 years. Find the money deposited and the interest rate.

Ques:3. If a person deposit ₹2000 in a saving account that earns interest at the rate of 6% p.a. compounded continuously, what is the value of the account at the end of 3 years.

Ques:4. A person deposited ₹4000 in a bank at 6% compounded continuously. After 3 years, the rate of interest was increased to 7 % and after 5 years, the rate was further increased to 8 %. The money was withdrawn at the end of 10 years. Find the amount.

Ques:5. A man made a deposit of ₹2500 in a saving account. The deposit was left to accumulate at 6% compounded quarterly for the first 5 years and at 8% compounded semiannually for the next 8 years.

Ques:6. Distinguish between the nominal and effective rate of interest. Also establish the relationship between nominal and effective rate of interest when compounded n times a year and when compounded continuously.

Ques:7. Find the effective rate equivalent to the nominal rate 6% converted (i) monthly, (ii) continuously.

Ques:8. Find, for each of the following, the amount to which ₹100 will accumulate:

- (i) At the rate of interest 12% p.a. compounded quarterly for 10 years.
- (ii) At the force of interest 3% p.a. for 3.5 years.
- (iii) At the effective rates of interest 3% p.a. for 10 years, 4% p.a. for 4 years and 5% p.a. for 2 years.
- (iv) At the rate of interest corresponding to 3% p.a. effective rate of discount for 8 years.
- (v) What constant force of interest would produce the same amount after 16 years as the rate in (iii) above.

Ques:9. Mr. Y has two investment options- either at 10% p.a. compounded semi-annually or 9.5% p.a. compounded continuously. Which option is preferable and why?

Ques:10. Mr. X left ₹50000 to be divided between his two daughters A and B. A's share was to amount to a certain sum of money at the end of 5 years and B's share was to amount to an equal amount at the end of 7 years. If the rate of interest is 6 % compounded annually, find the amount.

Ques:11. A debt of ₹5000 due 5 years from now and ₹5000 due 10 years from now is to be repaid by a payment of ₹2000 in 2 years, a payment of ₹4000 in 4 years and final payment at the end of 6 years. If the interest rate is 7% compounded annually, how much is final payment?

Ques:12. A person borrows ₹ 12,000. He pays ₹4000 at the end of 6 months and ₹5000 at the end of one year. What final payment should be made at the end of 2 years to settle the debt if the rate of interest is 12 % compounded semiannually.

Ques:13. A debt of ₹3000 which is due 6 years from now, is instead to be paid off by 3 payments ₹500 now, ₹1500 in 3 years and a final payment of ₹475 at the end of n years. If the rate of interest is 6 % p.a. effective, Find the value of n.

Ques:14. Mr. X took a loan of ₹ 50,000, payable with the interest at 10% p.a. compounded semiannually. If he pays ₹10,000 each at the end of first year and second year, find the balance payable at the end of third year if the rate of interest remains same.

Ques:15. The present value of ₹1000 due in 2 years at a certain nominal rate of discount, convertible semiannually, is ₹900. Find the rate of discount.

Ques:16. A computer whose cost is 10,00,000 will depreciate to a scrap value of 1,00,000 in 5 years. What is the book value of computer at the end of 4th year?

- (i) If the reducing balance method of depreciation is used.
- (ii) If the straight-line method of depreciation is used.

Ques:17. An asset costing ₹4500 will depreciate to a scrap value of ₹500 in 10 years. Find the rate of depreciate.

Ques:18. A machine costing ₹75000 is depreciated at the rate of 10% p.a. for the first 5 years and then at 12% p.a. for the next 3 year, both on diminishing balance basis. Find the book value at the end of 8th year. Using this, also find average rate of depreciation.

Questions for Annuity

Ques:1. At six-month interval, A deposited ₹2000 in a saving account which credit interest at 10% p.a. compounded semi-annually. The first deposit was made when A's son was six-month-old and the last deposit was made when his son was 8 years old. The money remained in the account and was presented to the son on his 10th birthday. How much did he receive?

Ques:2. An annuity of ₹500 p.a. is flowing continuously for 10 years. Find its future value if the rate of interest is 10% compounded continuously.

Ques:3. Mr. X deposits in his son's account ₹1000 times his son age at the end of each birthday. Find the balance accumulated at the 10th birthday, if the rate of interest is 10% p.a. compounded annually.

Ques:4. A man requires ₹ 2, 00,000 to purchase a house after 5 years. He has an opportunity to invest the fund in an account which can earn 6% p.a. compounded quarterly. Find how much be deposited at the end of each quarter so as to have the required amount at the end of 5 years.

Ques:5. Mr. X purchases a house for ₹2, 00,000. He agrees to pay for the house in 5 equal installments at the end of each year. If the money is worth 5% p.a. effective, what would be size of each investment? In case X makes a down payment of ₹50, 000 what would be the size of each installment?

Ques:6. What should be the monthly sales volume of a company if it desires to earn 12% annual returns convertible monthly on its investment of ₹ 2,00,000? Monthly costs are ₹3, 000. The investment will have eight-year life with no scrap value?

Ques:7. Mr. X sells his old car for ₹ 100,000 to buy a new one costing ₹ 2,58,000. He pays ₹ x cash and balance by payment of ₹7000 at the end of each month for 18 months. If the rate of interest is 9% compounded monthly, find x.

Ques:8. Find the capital value of a uniform income stream of ₹ R per year for m years, reckoning interest continuously at 100r% per year. What will be the result if income is forever?

Ques:9. According to an investment proposal, an initial investment of ₹ 1,00,000 is expected to yield a uniform income stream of ₹ 10,000 p.a. if the money is worth 8% p.a. compounded continuously, what is the expected payback period, i.e. after what time, the initial investment will be recovered?

Ques:10. If the present value and amount of an ordinary annuity of ₹1 p.a. for n years are ₹8.1109 and ₹12.0061 respectively, Find the rate of interest and the value of n without consulting the compound interest table.

Ques:11. Mr. X took a loan of ₹80,000 payable in 10 semiannual installments, rate of interest being 8% p.a. compounded semiannually, find:

- 1) The amount of each installment;
- 2) Loan outstanding after 4th payment;
- 3) Interest component of 5th payment; and
- 4) Loan repaid after four payments.

Ques:12. Mr. M borrowed ₹10, 00,000 from a bank to purchase a house and decided to repay by monthly equal installment in 10 years. The bank charges interest at 9% compounded monthly. The bank calculated his EMI as ₹12, 668. Find the principal and the interest paid in Ist and IInd year.

Ques:13. Machine A costs ₹ 10,000 and has a useful life of 8 years. Machine B costs ₹8000 and has a useful life of 6 years. Suppose machine a generates an annual labour saving of ₹ 2000 which machine B generate an annual saving of ₹1800. Assuming the time value of money is 10% p.a., find which machine is preferable?

Ques:14. Find the purchase of a ₹ 1000 bond, redeemable at the end of 10 years at ₹1100 and paying annual dividends at 4% if the yield rate is to be 5% p.a. effective.

Ques:15. A ₹1000 bond paying annual dividends at the 8.5% will be redeemed at par at the end of 10 years. Find the purchase price of this bond if the investor wishes a yield rate of 8%.

Ques:16. Mr. X wants ₹ 5,00,000 at the end of 7 years. If the rate of interest is 8%, what amount shall be deposited at the beginning of each quarter so as to get the above amount?

Ques: 17. Mr. X purchased an asset for ₹ 1,00,000 on installment basis. Each installment is to repaid at the beginning of each quarter. Find the size of each installment if the money is to be repaid in 3 years and the rate of interest is 6% compounded quarterly.

Ques: 18. X buys a piece of land for which he agrees to make 10 annual payments of ₹20, 000 each, the first being made at the end of 3 years. Find the equivalent cash price of this property if the money is worth 5% effective.

Ques: 19. A house sells for ₹ 50,000 down and 10 semiannual payments of ₹ 5000 each, the first due 3 years hence. Find the cash price of the house if the money is worth 6% compounded semiannually.

Ques: 20. How much is needed to endure a series of lectures costing ₹2500 at the beginning of each quarter of each year indefinitely, if the money is worth 3% compounded annually?

Ques: 21. Suppose a machine costing ₹70, 000 is to be replaced at the end of 5 years, at what time it will have a salvage value of ₹10,000. In order to provide money at that time for a new machine costing the same amount, sinking fund is set up. The amount in the fund at that time is to be the difference between the replacement costs and salvage value. If equal payments are placed in the fund at the end of each quarter and the fund earns 8% compounded quarterly, what each payment be?

Ques: 22. A machine costing ₹52, 000 and its effective life is estimated to be 12 years. A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap value is realized a sum of ₹5000 only. The price of a new model is estimated to be 25% higher than the price of a present one. Find what amount should be set aside at the end of each year, out of profits, for the sinking fund, if it accumulates at 10% effective.

Ques: 23. An income stream decreases continuously over time for x years, the income rate at t years from now being $₹ae^{-bt}$ per year. What is its present value if interest be reckoned at $100r\%$ compounded continuously? Show that this equals the capital value of a uniform income stream of ₹ a per year for x years if the rate of interest is raised to $100(r + b)\%$ per year.