GE-2: Linear Algebra (Semester II)
Vector Spaces

Note: Students are required to submit the hard copy of the solved assignment later on.

Q1. Mark each of the following statements TRUE or FALSE.

a) The set {0} is a subspace of V for any vector space V.
b) V is a subspace of itself for any vector space V.
c) $\mathbb{R}^3$ is a subspace of $\mathbb{R}^4$.
d) Let *W* is a subspace of *V*. If $u \in V$ and $\alpha.u \in W$ for all $\alpha \in \mathbb{R}$, then $u \in W$.
e) The set $\mathbb{Z} = \{ 0, \pm 1, \pm 2, ...\}$ of all integers is a vector space over $\mathbb{R}$ with usual addition and multiplication in $\mathbb{R}$.
f) The set $\mathbb{Q}$ of all rational numbers is a vector subspace of $\mathbb{R}$.
g) Every nonzero vector space *V* contains a nonzero proper subspace.

Q2. Let V be a vector space with dimension 12. Let S be a subset of V which is linearly independent and has 11 vectors.
State which of the following statements is TRUE or FALSE.

a) Every nonempty subset S1 of S is linearly independent.
b) S is a basis for V.
c) There must exist a linearly dependent subset S1 of V such that $S \subset S1$.
d) There must exist a linearly independent subset S1 of V such that $S \subset S1$ and S1 is a basis for V.
e) Dimension of span(S) < dimension of V.

Q3. Find an example of a subset of the vector space $\mathbb{R}$ that is closed under addition and contains the zero vector (which in this case is the number 0) but is not closed under scalar multiplication.

Q4. Show that the set $V = \{(x,y,z) \mid x,y,z \text{ in } \mathbb{R} \text{ and } x.x = z.z\}$ is not a subspace of $\mathbb{R}^3$.

Q5. Examine whether or not $M = \{(r,r+2,0) \mid r \text{ in } \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$

Q6. Let V be the vector space given by
$V = \mathcal{F}(\mathbb{R}, \mathbb{R})$ = Set of all real valued functions from $\mathbb{R}$ to $\mathbb{R}$
Then show that the set W,
$W = \mathcal{F}_e(\mathbb{R}, \mathbb{R})$ = Set of all even real valued functions from $\mathbb{R}$ to $\mathbb{R}$, is a subspace of V.

Q7. Let W_1 and W_2 be the subsets of $\mathbb{M}_{2x2}(\mathbb{R})$ given by
$W_1 = \{A \in \mathbb{M}_{2x2}(\mathbb{R}) \mid AX=0, X=\begin{bmatrix}1\\1\end{bmatrix}\}$ &
$W_2 = \{A \in \mathbb{M}_{2x2}(\mathbb{R}) \mid A^2=A\}$
Then check whether W_1 and W_2 are subspaces of $\mathbb{M}_{2x2}(\mathbb{R})$.

Q8. Let $P_k(x) = x^k + x^{k+1} + ... + x^n$, k=0,1,2,...,n.
Then show that the set $\{P_0(x), P_1(x),..., P_n(x)\}$ is linearly independent in $P_n(\mathbb{R})$.

Q9. Find a basis and dimension for the subspace W of $\mathbb{R}^4$ spanned by the set
$W = \{ [x, y, z, t] \mid x= y+z, z= y+t\}$

Q10. Find a basis for $\mathbb{R}^4$ that contains the vectors $v_1=[1, 0, 1, 0]$ and $v_2=[-1, 1, -1, 0]$.

Q11. Show that the set B
$B=\{ x^3+2x^2-4x+18, 3x^2+4x-4, x^3+5x^2-3, x+2\}$ is a basis for the vector space P_3.