B.A.(H) SECOND YEAR
Course9:Intermediate Microeconomics-II

EXTERNALITIES,PUBLIC GOODS,WELFARE and GENERAL EQUILIBRIUM

I.WELFARE and GENERAL EQUILIBRIUM

1(a) In an 'n' commodity economy, Walras' Law implies that (n-1) aggregate excess demand equations are independent.We also know that homogeneity of degree zero (in prices) of any aggregate excess demand function entails (n-1) independent prices.Why are these not sufficient for a competitive equilibrium to exist ?
(b) In the context of a competitive economy, the Brouwer's fixed point theorem consists of a function which maps the price simplex into the price simplex. What is this function called? Why must it be continuous? What are the assumptions which ensure its continuity?

2.Aafreen (A) and Brinda (B) consume apples (x) and oranges (y).Determine/show
(a) the Pareto set if their utility functions and endowments are represented by :
$u_A = \max [4x_A, y_A]$, $u_B = x_B + 2y_B]$, $w_A= (5,5)$ and $w_B= (5,5)$. Further, let 'B' be a monopolist and 'A' a price taking agent. Find the monopoly equilibrium allocation and the price ratio set by the monopolist.
(b) that $Px = P_y$ at the competitive equilibrium if their utility functions and endowments are:
$u_A= x_A y_A$, $u_B = \min [x_B ,y_B]$, the economy's endowment : w = (10,10)
(c) the competitive equilibrium if their utility functions and endowments are represented by :
$u_A = [(x_A)^2 + (y_A)^2]^{1/2}$ and $u_B =[(x_B)^2 + (y_B)^2]^{1/2}$ **,** $w_A= (5,5)$ and $w_B= (10,5)$. Is the competitive equilibrium allocation 'fair'?
(d) the competitive equilibrium if their utility functions and endowments are represented by :
$u_A = (x_A)^2 + (y_A)$ and $u_B = x_B . y_B$ **,** $w_A= (5,5)$ and $w_B= (10,5)$. What if $u_B = \min [x_B ,y_B]$?Does the First Welfare theorem hold in these two cases?

3(a) When is an allocation vector (i) envy free (ii) fair ?
Let the initial allocation in an economy be symmetric. Prove that the competitive market mechanism, in this economy, would always lead to a 'fair' allocation.
(b) Riyaz (A) and Yamini's(B) utility functions are defined over apples (x) and oranges (y) and are represented by $u_A(x_A,y_A) = x_A+2y_A^{1/2}$ and $u_B(x_B,y_B) = x_B + 4y_B^{1/2}$.The total endowments of 'x' and 'y'in the economy are(9,4).
(i) Find the equation of the UPF (Utility Possibility Frontier) in the (u_A, u_B) plane.
(ii) If the Social Welfare Function is represented by $W=u_A+u_B$,find the social welfare maximising allocation. Repeat when (a) $u_A =2(x_A.y_A)^{1/4}$, $u_B =(x_B.y_B)^{1/4}$ and (b) for an identical allocation for the 2 agents.
(c) Let a UPF be represented by $u_A^{1/2} +u_B =100$.If the SWF is 'weighted Benthamite', the welfare maximising allocation is $(u_A ,u_B) =(2500,50)$.Find the explicit form of the SWF.

4.Consider a pure exchange economy consisting of three individuals (A, B and C) with identical endowments of apples ('x') and oranges ('y') – each agent's allocation vector is (8,9). Their utility functions are represented by $u_A(x, y) = x_A y_A$, $u_B(x, y) = x_B\ y_B$ and $u_C(x, y) = x_C^{1/2} + y_C$ .Define a 'fair' allocation. Find a Pareto superior allocation to the endowments.Hence, show that these endowments do not represent a 'fair' allocation. Further , can you find an allocation in the **'core'** where A or C obtain zero units of 'x'?

EXTERNALITIES

5(a). Lavazza , a gourmet coffee manufacturer, roasts coffee in the basement of a building **owned** by it. Unroasted coffee beans cost Rs.200/kg and the **marginal** cost of roasting each kilogram of coffee is $150 – 10q + q^2$ (in Rupee terms).Roasted coffee sells at Rs.450 a kg. The smell of roasting , surprisingly, discomfits neighbours and they are willing to pay Rs $5q^2$ totally (where 'q'is the total kilograms roasted).
What is the Pareto efficient quantity of coffee roasted? The answer illustrates a theorem – state it and contextualize it in this problem.

(b)DDA plans to build an apartment complex next to Palam airport. Let 'x' represent the number of planes that land at Palam every day and let 'y' be the number of apartments in the housing complex. The profits of the airport are $42x – x^2$ and that of DDA are $42y–xy –y^2$.
(a) What would be the profit maximizing number of houses if DDA buys out Palam airport ?
(b) Let DDA and Palam airport operate as separate firms. What would be the profit maximizing number of houses made
-- if they operated as separate firms?
-- the government forces Palam to pay DDA an amount xy as 'damages'?

7. Assume that a steel firm, S, produces steel, s, and slag, x at Haridwar. The steel is sold in the marketplace while the slag gets dumped in the River Ganga. A fishery, F, located downstream at Allahabad is adversely affected by the slag released by the steel firm.
Let the unit price of fish be p_f and that of steel be p_s. Further, suppose the steel firm's cost function is
$c_s(s; x) = \alpha s^2 – \beta x + \gamma x^2$; $(\alpha,\beta,\gamma) > 0$

while the fishery's cost function is $c_f(f; x) = \delta f^2 + \theta x^2$; $(\delta,\theta) > 0$.

(a) Find the competitive profit maximizing quantities of steel ,fish and slag.
(b) If the 2 firms were to merge, what would be the quantities of steel ,fish and slag produced ?When would the Pareto efficient quantity of slag be less than that produced in the competitive equilibrium (in this question)?
(c) Assume the government imposes a tax on the slag generated by the steel firm. Calculate the Pigouvian tax that the government should impose so that the Pareto efficient quantity of slag is produced. Would it be the same as the price charged per unit of slag by the fishery if the property rights to clean water were vested in it?

PUBLIC GOODS

8.Ardashir (A), Bashir (B) hve utility functions defined over apples (a private good, 'x') and a flyover (a public good, 'F'). The utility function of an agent 'i' is defined by $u_i = 2 \log x_i + \log F$ where $F = F_A + F_B$ .Each agent has 100 apples as his endowment and 1 apple can be transformed into 1 flyover.
(a) What are the Nash equilibrium values of F_A and F_B ?
(b) What is the Pareto optimal level of F (don't use the Samuelson-Lindahl condition)? When would the Pareto optimal level of F not depend on the number of apples?
--What would be the consumption of apples by each agent if they contribute equally towards the public good (F)?
-- Would it change if an agent had a larger endowment of apples to begin with?

9.Ardashir (A), Bashir (B) and Diana (D) have utility functions defined over chocolates ('c'), a private good, and music ('M'), a **public good**. Their utility functions are defined by $u_i = c_i.M$, i= A,B and D. Chocolates (c) cost Re.1 each and music (M) costs Rs.10 an hour. The wealth (in Rs.) of agents A,B and D is Rs.30,Rs.50 and Rs.20 respectively.
Set out the Lagrangian to calculate the Pareto optimal quantity of music consumed. Confirm your calculations by writing out the Samuelson-Lindahl condtion for the Pareto optimal production of music (in this question).

10. In the case of discrete public goods, let the utility functions of the agents be defined by $u_A(x_A,G)$ and $u_B(x_B,G)$, where G can take only 2 values ,0 or 1.Let 'c' be the cost of one unit of G and g_A and g_B be A's and B's contribution to the public good. If r_A and r_B represent A's and B's reservation prices for G, derive the necessary and sufficient conditions for the provision of G to be Pareto improving. Would it still be Pareto superior to provide G if $r_A < g_A$?

ASYMMETRIC INFORMATION

11. Municipal Ward no.6 in Ranchi has a mix of 'healthy' and 'unhealthy' (morbid) people. A insurance company knows the exact number of 'healthy' and 'morbid' patients in the ward but not each patient's health history('type').The data on the number of healthy and unhealthy/morbid patients ,the 'willingness to pay' and the cost to the competitive insurance company are given in the following table :

	Number of Patients	Cost to the Insurance Co.	Patient's Willingness to Pay
Healthy	9000	1000	1200
Morbid	1000	5000	4000

(a) How would a full information equilibrium be characterised? What would be the price charged? At which price would both groups buy insurance?
(b) Would the market for health insurance (in this example) unravel in the presence of asymmetric information? Which economic term exemplifies this phenomenon?
Say, the government mandates that everyone must buy insurance and fixes the price of this 'compulsory' insurance at Rs 1200.
(c) What must be the lump sum subsidy that the government must pay per patient to the insurance company for the insurance company to break even?
(d) What is the consumer's surplus (CS) enjoyed by each 'morbid' consumer through this decree of the government? Calculate the social surplus (defined by the difference between the total consumers' surplus enjoyed by both groups and the subsidy paid by the government)?

12. Consider the market for 'second hand cars'. There are 1000 'second hand cars' for sale; a quarter are low quality ('lemons'), while the others are high quality ('plums'). The owner of a lemon is willing to sell it for any price above Rs.200, but the owner of a plum is willing to sell it for at least Rs.1100. Lemons are worth Rs.400 to buyers, and plums are worth Rs.1200 to them.
(a) If buyers cannot distinguish between the cars, how much would they be willing to pay for a car? Which cars get sold at equilibrium? Which economic term exemplifies this phenomenon? How does this equilibrium (p*,q*) compare with a full information equilibrium?

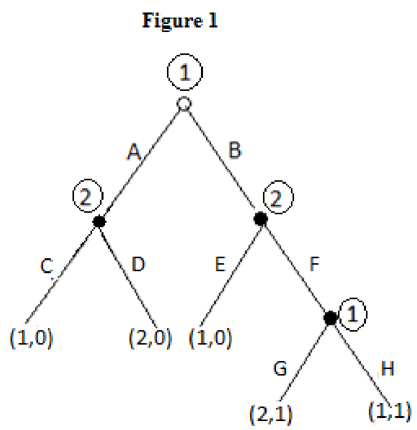
(b) Now assume that a fraction 'p' of the cars are plums. If buyers still cannot distinguish between cars, how large must 'p' be for all cars to be sold in equilibrium? Draw a diagram. What role can quality certification for cars play in this market? How much would the sellers of each type of car be willing to pay for a 'good quality' certification ?

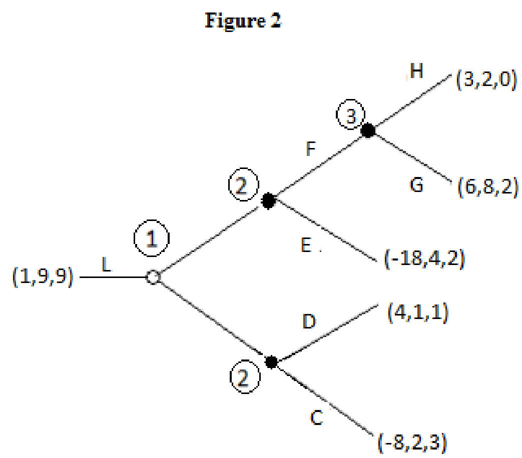
GAME THEORY

	L	C	R
T	**(3,2)**	**(1,0)**	**(1,1)**
M	**(0,1)**	**(2,3)**	**(1.5,1.5)**
B	**(1,1)**	**(0,0)**	**(0.7,0.7)**

13(a) Show that Player 1 has a strictly dominated strategy.Find all the pure strategy Nash equilibria of this game (if any exist).
(b) Let Player 2 play 'L' with probability 'q' and 'C' with probability '1–q'.Show that 'R' is strictly dominated by mixing 'L'and'C'.
(c) In the game which remains (after eliminating all strictly dominated strategies),find the mixed strategy Nash equilibrium. Draw the best response functions in the strategy space (p,q).

14.





14(a) For Figure 1, find
(i) all the Nash equilibria.
(ii)all the subgame perfect Nash equilibria. For any given Nash equilibrium that is not subgame perfect, explain how it exemplifies 'irrationality off the equilibrium path of play'?
(b) For **Figure 2**,enumerate :
(i)the number of subgames, the proper subhistories and the terminal histories.
(ii) the action set/s of each player after a given subhistory, the strategy set of each player and the outcome associated with any given strategy profile.
(iii) the SPNE of the game.

15. Let players 1 and 2 play a sequential game. Player 1 starts the game and can take one of 2 actions, A or B. If player 1 chooses A , player 2 can choose among actions C or D; should player 1 choose B , player 2 can choose between actions E or F. The payoff vector for actions A and C (by 1 and 2 respectively) is (3,0) [where the first element in the vector is the payoff to player 1 and the second element in the vector is the payoff to player 2]. The analogous payoff vectors associated with actions (A,D), (B,E) and (B,F) respectively are (2,0), (1,0) and (2,1) respectively.

Define a Subgame Perfect Nash Equilibrium in a sequential game. List out the strategies of players 1 and 2 to find all the Nash and Subgame Perfect Nash Equilibria of this game.

16. Consider a 3 agent example of Bertrand competition with search cost—a game played between 2 firms and a single consumer, Nisha. Each firm (i = 1,2) has 2 strategies – 'H' or 'L' – it can charge either Rs.3 or Rs.5. Nisha needs only 1 unit of good; the value (v) of the good for her is 6. The game is as follows:
--The two firms and the consumer play simultaneously: the firms set prices (H =5 or L=3) and Nisha decides whether to 'check' prices or 'not check' prices (thus her strategies are 'c' and 'n'). If she checks prices, she faces a search cost 's' , s ϵ (0,1). Her payoff is (v– Pi – s) if she checks prices and (v– P_i) if she does not.
-- If she checks the prices, then she buys from the firm with the lower price. If she decides not to check or if $P_1 = P_2$, then she buys from either of the firms with equal probabilities (and the firms get an expected payoff).
(a) Set this story up as a 3 person game (you will obtain 2 payoff matrices).

(b) Find the pure strategy Nash equilibrium/ equilibria of this game.

Problem Set 2-Gen Equilibrium and WelfareQuestions

1. Consider a pure exchange economy with three persons, 1, 2, 3 and two goods x and y. The utilities are given by $u^1(.) = xy, u^2(.) = x^3y, u^3(.) = xy^2$,respectively. If the endowments are (2,0), (0,12) and (12,0) respectively, then what can you say about the equilibrium price ratio?

2. Person A lexicographically prefers good x to good y. i.e., when comparing two bundles of x and y, she strictly prefers the bundle that has more of good x; if the bundles have equal amounts of good x, then she strictly prefers the bundle that has more of good y and person B considers x and y to be perfect substitutes, i.e., between bundles (x,y) and (x',y'), she strictly prefers (x,y) if and only if x+y>x'+y'. A's endowment is (0,1) and B's endowment is (2,0). Describe the competitive equilibrium in this economy.

3. Suppose that a city can be described by an interval [0,1]. Only 2 citizens, A and B, live in this city at different locations, A at 0.2 and B at 0.7. Government has decided to set up a nuclear power plant in this city but is yet to choose its location. Each citizen wants the plant as far as possible from her home and hence both of them have the same utility function, u(α)=α where α denotes the distance between the plant and home.

   a)Find the set of Pareto optimal locations for the plant. (Hint: Think about this hard, rather than solving this numerically)
   b)What is the social welfare maximizing outcome if the government follows the Classical welfare function ?Suppose there is another citizen C who is located at 0.5 and the government asks the citizens to vote between the Pareto optimal locations. What would be the outcome of majority voting? Which Pareto optimal location is the social welfare maximizing outcome in this case?

4. Consider a two person two good exchange economy: agents are A and B and goods are 1 and 2. The agents have the following utility functions:
   $$u_A(x_1, x_2) = \alpha x_1 + x_2 u_B(y_1, y_2) = y_1 y_2$$
   There are 5 units of each good. Consider the allocation where Agent A gets 4 units of good 1 only, but agent b gets 1 unit of good 1 and 5 units of good 2.
   a) For what values of α, is the above allocation a 'No-envy' allocation (no agent envies the other)?
   b) For what values of α, is the above allocation Pareto optimal?

5. Define the majority voting and rank-order voting rules. Using illustrations, describe the Condorcet's paradox of majority voting and violation of Independence of Irrelevant Alternatives property respectively. What is the major drawback associated with these voting rules?

Solve **all** the Varian workbook questions and try different combinations of the utility functions of the two agents in the exchange economy to find the competitive equilibrium, as already mentioned in the class.