Bertrand Oligopoly

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The Cournot oligopoly models a simultaneous move game in which firms choose their quantities. For the total quantity produced, the inverse demand function gives the market price. In the *Bertrand oligopoly*, each firm chooses the *price* of its output and it is assumed that it satisfies all the demand it faces at that price. It is also assumed that the firm that chooses the lowest price gets all the market demand, which it is able to satisfy, while every other firm faces zero demand for its output. If the lowest price is charged by two or more firms, then the market is divided equally among these firms while each of the rest of the firms gets zero market share. If p is the lowest price charged by only one firm, then $D(p)$ is the quantity demanded at that price, where $D(.)$ is the demand function faced by the industry. If m firms are charging the lowest price p, and every other firm charging a price higher than p, then the industry demand $D(p)$ is split equally between these m firms, i.e., each firm faces the demand $D(p)/m$.

The General Model

The Bertrand Oligopoly is the following simultaneous move game.

The n firms are the *players*.

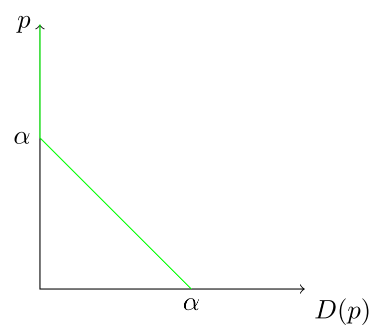
The set of strategies of each firm i is the set of possible prices ($p_i \geq 0$).

If firm i is the only firm choosing the lowest price p_i, then its total profit is $p_i D(p_i) - C_i(D(p_i))$ where $C_i(q_i)$ is the total cost of producing q_i units of output. If i is one of the m firms choosing the lowest price p_i, then its profit is $[p_i D(p_i/m) - C_i(D(p_i/m))]$. **Note that $m = 1$ if firm i's price p_i is lower than every other price**. Firm i's profit is zero if there is some other price lower than p_i.

Example: duopoly with constant unit cost and linear demand function

Suppose there are two firms, 1 and 2, each of whose cost function has constant unit cost c, such that $C_i(q_i) = cq_i$ for $i = 1, 2$. Assume that the demand function is

$$D(p) = \begin{cases} \alpha - p & \text{if } p \leq \alpha \\ 0 & \text{if } p > \alpha \end{cases}$$



If firm i chooses the lowest price p_i, then it gets all the market demand $D(p_i)$ and its profit is $p_iD(p_i) - cD(p_i) = p_i(\alpha - p_i) - c(\alpha - p_i) = (p_i - c)(\alpha - p_i)$. If firms i, j ($j = 2$ if $i = 1$ and $j = 1$ if $i = 2$) both charge the same price, then firm i's profit (= firm j's profit) is $p_iD(p_i)/2 - cD(p_i)/2 = p_i(\alpha - p_i)/2 - c(\alpha - p_i)/2 = [(p_i - c)(\alpha - p_i)]/2$. If firm i charges higher price then it's profit is zero since firm j faces all the market demand $D(p_j)$.

$$\Pi_i(p_i, p_j) = \begin{cases} (p_i - c)(\alpha - p_i) & \text{if } p_i < p_j \\ \frac{1}{2}(p_i - c)(\alpha - p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

Claim: $(p_1, p_2) = (c, c)$ is a Nash equilibrium. No other pair (p_1, p_2) is a Nash equilibrium.

Homework Assignment[1]

1. Prove the above claim. Argue why (p_1, p_2) is the only Nash equilibrium.
2. (Bertrand oligopoly) Consider the model with $n \geq 3$ firms. Show that any profile $(p_1, p_2, ..., p_n)$ of prices for which $p_i \geq c$ for all i and atleast two prices are equal to c is a Nash equilibrium and no other profile is a Nash equilibrium.

[1]Submit by April 2, 2020, 10 pm IST at the email-id provided.